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IIB-branes and new spacetime superalgebras

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Abstract: We provide a classification of the IIB Dp- and NSp-branes in which the brane action exists due to a non-trivial class of the Chevalley-Eilenberg cohomology of free differential algebras. We then present a new geometric formulation of the IIB Dp- and NSp-branes ($p \leq 5$) in which the manifestly superinvariant Wess-Zumino terms are constructed in terms of the supersymmetric currents. The supercurrents are obtained by using supergroup manifolds corresponding to the IIB-brane superalgebra, which is characterized by the generators of D3-, D5-, NS5- and KK5-branes in addition to the previously introduced generators of supertranslations, F- and D-strings. The charges of D1-, F1- and D3-branes are related to those of the M-algebra, but some charges of D5- and NS5-branes are not. The S-duality of the type-IIB theory is realized as transformations of the supercurrents generalizing the SO(2) R-symmetry of the superalgebra. We thus find that the superalgebra is lifted into twelve-dimensions with signature (11,1).

Keywords: p-branes, D-branes, String Duality, Superspaces.

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1. Introduction

It is now widely appreciated that super p-branes play an important role in studying non-perturbative superstring physics [1, 2]. The existence of a p-brane is guaranteed by the existence of the corresponding Wess-Zumino (WZ) term, which is needed for the κ-symmetry, or the matching of the bosonic and fermionic degrees of freedom on the worldvolume [3]. The WZ term is determined by a closed $(p+2)$-form which is shown to be characterized by non-trivial Chevalley-Eilenberg $(p+2)$-cocycles on the ordinary superspace [4]. These WZ term was shown to cause topological charges and modify the super Poincaré algebra [5]. In the spacetime superalgebra, p-branes are expressed as p-form "central" charges [6, 7, 8, 9].

It is known that the WZ action $I_{\rm WZ} = \int \mathcal{L}_{\rm WZ}$ is superinvariant, but the integrand $\mathcal{L}_{\rm WZ}$ is not. The WZ term $\mathcal{L}_{\rm WZ}$ is superinvariant up to a total derivative term. A manifestly supersymmetric formulation of the Green-Schwarz superstring, including the WZ term, was given by Siegel [10] based on a superalgebra [11] which is the global limit of a superalgebra [12] found by himself. The superalgebra is a generalization of super Poincaré algebra and generated by generators of supertranslations (P_a, Q_α) and those of F-strings (Z^a, Z^α), where Z^α is a new fermionic generator. Constructing a suitable set of supercurrents on the corresponding supergroup manifold, he wrote down the WZ term of the Green-Schwarz superstring in a second-order expression of the supercurrents. Using this formulation, p-branes $(p > 1)$, found in the brane scan [3], were formulated by Bergshoeff and Sezgin [13]. Introducing the p-brane superalgebras generated by generators of supertranslation and those of p-brane $(Z^{a_1\cdots a_p}, \ldots, Z^{\alpha_1\cdots\alpha_p})$, they wrote down the manifestly supersymmetric WZ terms for the p-branes, which are $(p+1)$-th order in the supercurrents (further discussions on the superalgebra are found in [14]). For the non-minimal type-II theories, type-II superstrings and IIB D-strings [15], IIB (p,q)-strings [16, 17] and IIA D2-brane [18] are formulated by introducing the corresponding superalgebras. In [15], it was pointed out that worldvolume gauge fields can be expressed in terms of coordinates corresponding to new generators. This was further investigated in [17, 18].

In this paper, we generalize them to incorporate all the branes in the type-IIB theory. The type-IIB theory is chiral with $16+16$ Majorana-Weyl supercharges with the same chirality. The 528 components of the anti-commutator of the supercharges are distributed as

$$\mathbf{528} = \mathbf{10} + \mathbf{10} + \mathbf{10} + \mathbf{120} + \mathbf{126} + \mathbf{126} + \mathbf{126}\,,$$

among the components of generators: translations, D-strings (D9-branes), F-strings (Neveu-Schwarz (NS) 9-branes), D3-branes (D7- and NS7-branes), D5-branes, NS5-branes and Kaluza-Klein (KK) 5-branes, respectively, where branes in the parentheses correspond to charges with a time index. It is shown that the WZ terms for Dp- and NSp-branes are characterized by Chevalley-Eilenberg (CE) cohomology on the ordinary superspace. The obtained WZ terms are equivalent to those obtained in [19], and the total actions are invariant under the κ-symmetry. In order to formulate the WZ action to be manifestly superinvariant, we introduce new generators with spinorial indices in exchange for Lorentz indices of bosonic brane charges of the superalgebra with the maximal "central" extension, and provide new spacetime superalgebra, the IIB-brane superalgebra. The IIB-brane superalgebra contains various subalgebras found before: $N = 1$ p-brane superalgebra [10, 13], the IIB-superstring superalgebra [15], etc. The D1, F1, D3-brane and KK5-brane charges are obtained by a T-duality from those in the M-algebra found in [20], but the D5- and NS5-brane charges with five spinorial indices are not. In this sence, the IIB-brane superalgebra is a generalization of the M-algebra. By using the IIB-brane superalgebra, we show that the IIB Dp- and NSp-branes ($p \leq 5$) can be formulated in a manifestly superinvariant way. Namely, the WZ terms are found to be written down in terms of the supercurrents on the supergroup manifold corresponding to the superalgebra. New coordinates associated with new generators are contained in a surface term, by virtue of the fiber bundle structure of our superalgebra with the ordinary superspace being the base space. The WZ actions of the superinvariant WZ terms are equivalent to those obtained in the CE cohomology classification. Thus, the total actions are κ-invariant. The IIB-brane superalgebra enjoys the SO(2) symmetry, which is a generalization of the SO(2) R-symmetry, and thus the S-duality is generalized to include new generators. The S-duality is shown to be realized geometrically in twelve-dimensions.

This paper is organized as follows. We begin with showing that the IIB WZ terms are characterized by the CE cohomology in section 2. We introduce two free differential algebras, which turn out to correspond to D-branes and NS-branes. The obtained WZ terms are shown to be equivalent to those of the κ-invariant D/NSp-brane actions found before. In section 3, introducing a set of new spacetime superalgebras, we construct the superinvariant WZ terms for D- and NS-branes using the supercurrents on the corresponding group manifolds. The properties of superalgebras and the relations to the M-algebra are discussed in section 4. It is shown that this IIB-brane superalgebra enjoys SO(2) symmetry. The superalgebra is found to be lifted into twelve-dimensions with signature (11,1), while realizing the SO(2) symmetry geometrically, in section 5. The last section is devoted to a summary and discussions.

2. Chevalley-Eilenberg cohomology classification of IIB-branes

The WZ term $\mathcal{L}_{\rm WZ}$ is known to be superinvariant up to a total derivative term. It follows that $(p+2)$-form $h_{p+2} \equiv d\mathcal{L}_{\rm WZ}$ is superinvariant. For the p-branes [4] and the IIA Dp-branes [18], the $(p+2)$-form h_{p+2} were characterized by being a non-trivial CE cohomology $(p+2)$-cocycles on superspace. In this section, we give a classification of the IIB Dp-branes and the IIB NSp-branes using the CE cohomology [21].

For the type-IIB theory, the Poincaré superalgebra is generated by the translation generator P_a and $16+16$ Majorana-Weyl supercharges $(Q_\alpha, Q_{\tilde{\alpha}})$ with the same chirality, as[1]

$$\{Q_{\boldsymbol{\alpha}}, Q_{\boldsymbol{\beta}}\} = (\gamma^a 1)_{\boldsymbol{\alpha\beta}} P_a\,, \qquad [P_a, Q_{\boldsymbol{\alpha}}] = [P_a, P_b] = 0\,, \tag{2.1}$$

where γ^a's are the ten-dimensional γ-matrices satisfying $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and 1 acts on a column vector $Q_{\boldsymbol{\alpha}} = \binom{Q_{\tilde{\alpha}}}{Q_\alpha}$. The left-invariant (LI) Maurer-Cartan one-form Π^A is defined by $\Pi^A T_A = U^{-1} dU$, where U is a group element and T^A collectively denotes the generators of the algebra. Let $(x^a, \theta^{\boldsymbol{\alpha}})$ be the coordinates on the corresponding group manifold, the LI one-form is expressed as $\Pi^a = dx^a + \frac{1}{2}(\bar{\theta}\gamma^a 1 d\theta)$ and $\Pi^{\boldsymbol{\alpha}} = d\theta^{\boldsymbol{\alpha}}$ and provides a basis for the differential forms on the group manifold. The LI group vielbein $L_M{}^A$ is read off from $\Pi^A = dZ^M L_M{}^A$, where Z^M denotes coordinates on the supergroup manifold. Similarly, the right-invariant (RI) Maurer-Cartan one-form Ξ^A is defined by $\Xi^A T_A = dU U^{-1}$, and the RI group vielbein $R_M{}^A$ is read off from $\Xi^A = dZ^M R_M{}^A$. The RI generators Q_A of the left-translation, written as $Q_A = R_A{}^M \partial_M$, are regarded as the supertranslation generators, while the LI generators D_A of the right-translation, written as $D_A = L_A{}^M \partial_M$, are regarded as the supercovariant derivatives. The LI one-form Π^A is invariant under the supertranslations.

In order to classify the IIB Dp-branes, we introduce two-form $\mathcal{F}$, which is the modified field strength of the NS$\otimes$NS two-form gauge potential as well as the LI one-form Π^A. The non-trivial $(p+2)$-cocycles are given by closed $(p+2)$-forms $h_{p+2}(\Pi^a, \Pi^\alpha, \Pi^{\tilde{\alpha}}, \mathcal{F})$ which cannot be written as the differential of a $(p+1)$-form constructed from $\Pi^a, \Pi^\alpha, \Pi^{\tilde{\alpha}}$ and $\mathcal{F}$. The possible WZ terms will be characterized as some non-trivial $(p+2)$-cocycles of the cohomology of a certain free differential algebra (FDA).

Let us consider the FDA $\mathfrak{F}$ generated by $\Pi^a, \Pi^\alpha, \Pi^{\tilde{\alpha}}, \mathcal{F}$ and defined by the structure relations:

$$d\Pi^a = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma^a 1)_{\boldsymbol{\alpha\beta}}\,, \tag{2.2}$$

$$d\Pi^{\boldsymbol{\alpha}} = 0\,, \tag{2.3}$$

$$d\mathcal{F} = \Pi^a \Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a \sigma_3)_{\boldsymbol{\alpha\beta}}\,. \tag{2.4}$$

The wedge product $\wedge$ is to be understood here and henceforth. Given a super q-form G, the exterior derivative acts as follows: $d(F \wedge G) = F \wedge dG + (-1)^q dF \wedge G$. The nilpotency $dd = 0$ is satisfied by the Fierz identity: $(\gamma_a 1)_{(\boldsymbol{\alpha\beta}}(\gamma^a\sigma_3)_{\boldsymbol{\gamma\delta})} = 0$. The superinvariant closed $(p+2)$-form can be expanded in terms of the two-form $\mathcal{F}$ as

$$h_{p+2} = \sum_{n=0}^{[(p+2)/2]} \frac{1}{n!} h^{(p+2-2n)}(\Pi^a, \Pi^{\boldsymbol{\alpha}})\mathcal{F}^n\,, \tag{2.5}$$

[1]The notation and conventions are similar to [6]. The vector indices are $a = 0, 1, \ldots, 9$ and the spinor indices are $\boldsymbol{\alpha}, \boldsymbol{\beta}$. The charge conjugation matrix is $c_{\alpha\beta}$ and $\gamma^{a_1\cdots a_n} = \gamma^{[\alpha_1}\cdots\gamma^{a_n]}$.

where $h^{(p+2-2n)}$ is a $(p+2-2n)$-form written in terms of Π^a and $\Pi^{\boldsymbol{\alpha}}$. Assigning dimension $[\Pi^a] = L$, $[\Pi^\alpha] = L^{1/2}$, $[\mathcal{F}] = L^2$, $[h_{p+2}] = L^{p+1}$, to avoid introducing extra dimensionful constants, one writes the k-form $h^{(k)}$ with dimension L^{k-1} as

$$h^{(k)} = a^{(k)} \Pi^{a_1} \cdots \Pi^{a_{k-2}} \Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} \left(\gamma_{a_1 \cdots a_{k-2}} \sigma^{(k)} \right)_{\boldsymbol{\alpha\beta}} , \tag{2.6}$$

where $a^{(k)}$ is a constant and $\sigma^{(k)}$ is a 2×2 matrix determined below. Note that $\Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} (\gamma_{a_1 \cdots a_{k-2}} \sigma^{(k)})_{\boldsymbol{\alpha\beta}}$ is non vanishing when

$$\begin{aligned} \sigma^{(k)} &= 1 \,, \sigma_3, \sigma_1 && \text{for } k-2 = 1 \text{ mod } 4 \,, \\ \sigma^{(k)} &= i\sigma_2 && \text{for } k-2 = 3 \text{ mod } 4 \,. \end{aligned} \tag{2.7}$$

This immediately implies that there are no non-trivial $(p+2)$-cocycles when $p =$ even. This is consistent with the fact that the IIB D- and NS p-branes exist only when $p =$odd. Thus, the closedness condition $dh_{p+2} = 0$ reduces to conditions

$$dh^{(1)} = 0 \,, \qquad dh^{(k)} + h^{(k-2)} d\mathcal{F} = 0 \,, \quad k = 3, 5, 7, \ldots, p+2 \,. \tag{2.8}$$

As a result, we find that the non-trivial h_{p+2} is constructed on the FDA $\mathfrak{F}$ in terms of

$$h^{(1)} = 0 \,, \tag{2.9}$$

$$h^{(3)} = a \Pi^a \Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} (\gamma_a \sigma_1)_{\boldsymbol{\alpha\beta}} \,, \tag{2.10}$$

$$h^{(5)} = a \frac{2}{3!} \Pi^{a_1} \Pi^{a_2} \Pi^{a_3} \Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} (\gamma_{a_1 a_2 a_3} i\sigma_2)_{\boldsymbol{\alpha\beta}} \,, \tag{2.11}$$

$$h^{(7)} = a \frac{2^2}{5!} \Pi^{a_1} \cdots \Pi^{a_5} \Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} (\gamma_{a_1 \cdots a_5} \sigma_1)_{\boldsymbol{\alpha\beta}} \,, \tag{2.12}$$

$$h^{(9)} = a \frac{2^3}{7!} \Pi^{a_1} \cdots \Pi^{a_7} \Pi^{\boldsymbol{\alpha}} \Pi^{\boldsymbol{\beta}} (\gamma_{a_1 \cdots a_7} i\sigma_2)_{\boldsymbol{\alpha\beta}} \,, \tag{2.13}$$

where a is a normalization constant to be set being 1 below. In the course to obtain the above results we used Fierz identities. In $(9+1)$-dimensions, letting $\phi_i, i = 1, 2, 3, 4$, be Grassmann even Majorana-Weyl spinors with the same chirality, Fierz identity is expressed as

$$\left(\phi_1 \Lambda^1 \phi_2 \right) \left(\phi_3 \Lambda^2 \phi_4 \right) = \frac{1}{16} \sum_A \left(\phi_1 \mathcal{O}^A \phi_4 \right) \left(\phi_3 \Lambda^2 \mathcal{O}^A \Lambda^1 \phi_2 \right) , \tag{2.14}$$

$$\mathcal{O}^A = \left\{ \gamma^a, \frac{i}{\sqrt{3!}} \gamma^{a_1 a_2 a_3}, \frac{1}{\sqrt{2 \cdot 5!}} \gamma^{a_1 \cdots a_5} \right\}. \tag{2.15}$$

In addition, a useful relation is

$$\gamma^{a_1 \cdots a_N} = \frac{-(-1)^{N(N-1)/2}}{(10-N)!} \epsilon^{a_1 \cdots a_N b_1 \cdots b_{10-N}} \gamma_{b_1 \cdots b_{10-N}} \gamma^{\natural} \,, \tag{2.16}$$

where $\gamma^\natural = \gamma^{01\cdots 9}$. In particular, h_{p+2} is closed due to the Fierz identities for Dp-branes:

$$\text{D-string}: \ (\gamma_a)_{(\alpha\beta}(\gamma^a)_{\gamma)\tilde{\delta}} = 0\,, \tag{2.17}$$
$$\text{D3-brane}: \ (\gamma_c)_{(\alpha\beta}\left(\gamma^{abc}\right)_{\gamma)\tilde{\delta}} + 2\left(\gamma^{[a}\right)_{(\alpha\beta}\left(\gamma^{b]}\right)_{\gamma)\tilde{\delta}} = 0\,, \tag{2.18}$$
$$\text{D5-brane}: \ (\gamma_b)_{(\alpha\beta}\left(\gamma^{a_1\cdots a_4 b}\right)_{\gamma)\tilde{\delta}} + 4\left(\gamma^{[a_1}\right)_{(\alpha\beta}\left(\gamma^{a_2a_3a_4]}\right)_{\gamma)\tilde{\delta}} = 0\,, \tag{2.19}$$
$$\text{D7-brane}: \ (\gamma_b)_{(\alpha\beta}\left(\gamma^{a_1\cdots a_6 b}\right)_{\gamma)\tilde{\delta}} + 6\left(\gamma^{[a_1}\right)_{(\alpha\beta}\left(\gamma^{a_2\cdots a_6]}\right)_{\gamma)\tilde{\delta}} = 0\,. \tag{2.20}$$

The (anti-) symmetrization with a unit weight of indices in parentheses (square brackets) is understood. For D9-branes, the Fierz identity is

$$\text{D9-brane}: \ (\gamma_b)_{(\alpha\beta}\left(\gamma^{a_1\cdots a_8 b}\right)_{\gamma)\tilde{\delta}} + 8\left(\gamma^{[a_1}\right)_{(\alpha\beta}\left(\gamma^{a_2\cdots a_8]}\right)_{\gamma)\tilde{\delta}} = 0 \tag{2.21}$$

while $h^{(11)}$ is formally eleven-dimensional. $h^{(11)}$ is characterized by $a^{(11)} = (2^4/9!)a$ and $\sigma^{(11)} = \sigma_1$. The Fierz identities (2.20) and (2.21) are obtained from (2.19) and (2.18), respectively. One finds that the Fierz identities for Dp-brane are collectively expressed as

$$(\gamma_b)_{(\alpha\beta}(\gamma^{a_1\cdots a_{p-1} b})_{\gamma)\tilde{\delta}} + (p-1)\left(\gamma^{[a_1}\right)_{(\alpha\beta}\left(\gamma^{a_2\cdots a_{p-1}]}\right)_{\gamma)\tilde{\delta}} = 0\,. \tag{2.22}$$

In order to see that the obtained $(p+2)$-forms are not CE cohomology trivial, we note that if there were $(p+1)$-forms $b(\Pi^a, \Pi^{\boldsymbol{\alpha}}, \mathcal{F})$ such that $db = h_{p+2}$, then b's would be Lorentz invariant $(p+1)$-forms with dimensions $[b] = L^{p+1}$. One sees that such a b does not exist.

D-brane actions, which are κ-invariant, have been proposed in [22, 19, 23, 24, 25, 26]. We show that our action is κ-invariant by relating our action to the κ-invariant action of [19]. Rescaling supercurrents as $\Pi^a \to k\Pi^a$, $\Pi^{\boldsymbol{\alpha}} \to l\Pi^{\boldsymbol{\alpha}}$, where $k = 1/\sqrt{2}$ and $kl^2 = 1$, (2.2) is transformed into $\Pi^a = -\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a 1)_{\boldsymbol{\alpha\beta}}$ and (2.3) and (2.4) are unchanged. These are normalizations of supercurrents used in [19]. In this normalization, numerical coefficients of $h^{(k)}, k = 3, 5, 7, 9, 11$, are expressed as $1, 1/3!, 1/5!, 1/7!, 1/9!$, respectively. One find that the $(p+2)$-forms h_{p+2} constructed from these $h^{(k)}$ are identical to those of WZ terms obtained in [19]. It follows that, introducing the Dirac-Born-Infeld action I_{DBI} as (3.1), the combined actions, $I = I_{\mathrm{DBI}} + I_{\mathrm{WZ}}$, are shown to be κ-invariant as was done there, but we do not repeat this here. In this way, we find that κ-invariant IIB Dp-brane actions are characterized by using the CE cohomology. In order to classify the WZ terms for NSp-branes, we consider the FDA $\mathfrak{G}$ generated by $\Pi^a, \Pi^{\boldsymbol{\alpha}}, \mathcal{G}$ and defined by the structure relations

$$d\Pi^a = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\,\Pi^{\boldsymbol{\beta}}(\gamma^a 1)_{\boldsymbol{\alpha\beta}}\,, \tag{2.23}$$
$$d\Pi^{\boldsymbol{\alpha}} = 0\,, \tag{2.24}$$
$$d\mathcal{G} = -\Pi^a\,\Pi^{\boldsymbol{\alpha}}\,\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_1)_{\boldsymbol{\alpha\beta}}\,, \tag{2.25}$$

where $\mathcal{G}$ turns out to be the modified field strength of the Ramond$\otimes$Ramond (R$\otimes$R) two-form gauge potential. The closed $(p+2)$-form is expanded as $\hat{h}_{p+2} = \sum(1/n!)\hat{h}^{(p+2-2n)}(\Pi^a, \Pi^{\boldsymbol{\alpha}})\mathcal{G}^n$ where $\hat{h}^{(k)}$ is expressed as $\hat{h}^{(k)} = b^{(k)}\Pi^{a_1}\cdots\Pi^{a_{k-2}}\Pi^{\boldsymbol{\alpha}}\wedge \wedge\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_{k-2}}\sigma^{(k)})_{\boldsymbol{\alpha\beta}}$. Solving $d\hat{h}_{p+2} = 0$:

$$d\hat{h}^{(1)} = 0\,, \qquad d\hat{h}^{(k)} + \hat{h}^{(k-2)}d\mathcal{G} = 0\,, \quad k = 2, 3, 7, \ldots, p+2\,, \tag{2.26}$$

we find

$$\hat{h}^{(1)} = 0\,, \tag{2.27}$$
$$\hat{h}^{(3)} = b\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_3)_{\boldsymbol{\alpha\beta}}\,, \tag{2.28}$$
$$\hat{h}^{(5)} = b\frac{2}{3!}\Pi^{a_1}\cdots\Pi^{a_3}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_3}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \tag{2.29}$$
$$\hat{h}^{(7)} = b\frac{2^2}{5!}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_5}\sigma_3)_{\boldsymbol{\alpha\beta}}\,, \tag{2.30}$$
$$\hat{h}^{(9)} = b\frac{2^3}{7!}\Pi^{a_1}\cdots\Pi^{a_7}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_7}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \tag{2.31}$$

where b is a constant to be set being 1 below. The $(p+2)$-forms $\hat{h}_{p+2}$ constructed from these $\hat{h}^{(k)}$'s are shown to be CE cohomology non trivial, as before. As will be seen in the next section, $\hat{h}_{p+2}$ is related to h_{p+2} by the S-duality. We regard the branes associated to the WZ terms characterized by the above as the IIB NS-branes. For the 3-brane, we call it D3-brane since the WZ term is transformed to that of the D3-brane by a vector-vector duality on the worldvolume gauge field (see section 3.2). The 7-branes will be regarded as the NS7-branes.

$\hat{h}_{p+2}$ is closed due to Fierz identities:

$$\text{F-string}: \quad (\gamma_a)_{(\alpha\beta}(\gamma^a)_{\gamma\delta)} = 0\,, \tag{2.32}$$

$$\text{D3-brane}: \quad (\gamma_c)_{(\alpha\beta}(\gamma^{abc})_{\gamma)\tilde{\delta}} + 2\left(\gamma^{[a}\right)_{(\alpha\beta}\left(\gamma^{b]}\right)_{\gamma)\tilde{\delta}} = 0\,, \tag{2.33}$$

$$\text{NS5-brane}: \begin{cases} (\gamma_b)_{(\alpha\beta}(\gamma^{a_1\cdots a_4 b})_{\gamma\delta)} = 0\,, \\ (\gamma_b)_{\alpha\beta}(\gamma^{a_1\cdots a_4 b})_{\tilde{\gamma}\tilde{\delta}} - (\gamma_b)_{\tilde{\gamma}\tilde{\delta}}(\gamma^{a_1\cdots a_4 b})_{\alpha\beta} + 16(\gamma^{[a_1})_{\alpha\tilde{\gamma}}(\gamma^{a_2 a_3 a_4]})_{\beta\tilde{\delta}} = 0\,, \end{cases} \tag{2.34}$$

$$\text{NS7-brane}: (\gamma_b)_{(\alpha\beta}(\gamma^{a_1\cdots a_6 b})_{\gamma)\tilde{\delta}} + 6\left(\gamma^{[a_1\cdots a_5}\right)_{(\alpha\beta}\left(\gamma^{a_6]}\right)_{\gamma)\tilde{\delta}} = 0\,. \tag{2.35}$$

In the second line of (2.34), the symmetrization of (α,β) and that of $(\tilde{\gamma},\tilde{\delta})$ are understood. Note that (2.20) is different from (2.35) while $h^{(9)}$ is the same as $\hat{h}^{(9)}$. The identity (2.35) is obtained from (2.19) and the first line of (2.34).

For NS9-branes, the Fierz identity is

$$\text{NS9-brane}: \begin{cases} (\gamma_b)_{(\alpha\beta}(\gamma^{a_1\cdots a_8 b})_{\gamma\delta)} = 0\,, \\ (\gamma_b)_{\alpha\beta}\left(\gamma^{a_1\cdots a_8 b}\right)_{\tilde{\gamma}\tilde{\delta}} - (\gamma_b)_{\tilde{\gamma}\tilde{\delta}}\left(\gamma^{a_1\cdots a_8 b}\right)_{\alpha\beta} + 32\left(\gamma^{[a_1}\right)_{\alpha\tilde{\gamma}}\left(\gamma^{a_2\cdots a_8]}\right)_{\beta\tilde{\delta}} = 0\,, \end{cases} \tag{2.36}$$

while $h^{(11)}$ is formally eleven-dimensional. One finds that $h^{(11)}$ is characterized by $b^{(11)} = 2^4/9!b$ and $\sigma^{(11)} = \sigma_3$. The second line of (2.36) is obtained from

$$\left(\gamma^{[a}\right)_{\alpha\beta}\left(\gamma^{b]}\right)_{\tilde{\gamma}\tilde{\delta}} + (\gamma^c)_{\alpha\tilde{\gamma}}(\gamma^{ab}{}_c)_{\beta\tilde{\delta}} = 0\,, \tag{2.37}$$

where the symmetrization of (α, β) and that of $(\tilde{\gamma}, \tilde{\delta})$ are understood. We find that the Fierz identities for NS/Dp-branes are collectively expressed as

$$\begin{aligned}&\left(\gamma^{a_1\cdots a_{p-1}b} i\sigma_2 \tau^{(p+1)/2}\right)_{(\boldsymbol{\alpha\beta}}\left(\gamma^b 1\right)_{\boldsymbol{\gamma\delta})} + \\ &\qquad + (p-1)\left(\gamma^{[a_1\cdots a_{p-2}} i\sigma_2 \tau^{(p-1)/2}\right)_{(\boldsymbol{\alpha\beta}}\left(\gamma^{a_{p-1}]}\tau\right)_{\boldsymbol{\gamma\delta})} = 0\,,\end{aligned} \tag{2.38}$$

where $\tau = \sigma_1$ for NSp-branes and $\tau = \sigma_3$ for Dp-branes. In this way, we have classified the WZ terms for NSp-branes. Denoting the obtained WZ action as $\hat{I}_{\rm WZ}$ and introducing $\hat{I}_{\rm DBI}$ as (3.7), one expects that the combined actions, $\hat{I} = \hat{I}_{\rm DBI} + \hat{I}_{\rm WZ}$, are shown to be κ-invariant. In fact, $\hat{I}$ can be rewritten as the κ-invariant actions I by the redefinition of supercurrents: $\Pi^{\boldsymbol{\alpha}} \to \sigma\Pi^{\boldsymbol{\alpha}}$, where $\sigma = \pm 1/\sqrt{2}(1 + i\sigma_2)$, as will be seen in section 3. We expect that the same procedure employed in showing the κ-invariance of Dp-branes will work here.

We have classified closed superinvariant $(p+2)$-forms h_{p+2} and $\hat{h}_{p+2}$, and then the WZ term $\mathcal{L}_{\rm WZ}$ for Dp-branes and $\hat{\mathcal{L}}_{\rm WZ}$ for NSp-branes, respectively. In the next section, we construct the WZ terms for Dp- and NSp-branes ($p \leq 5$) in a manifestly supersymmetric way.

3. Supersymmetric Wess-Zumino terms

We construct the manifestly superinvariant WZ terms using the supercurrents on the supergroup manifolds of the corresponding superalgebras. We discuss the relations of these superalgebras to the M-algebra and the IIA superalgebra in the next section.

The Dp-brane action is composed of the Dirac-Born-Infeld (DBI) action and the WZ action. The DBI action is (background scalars are omitted)

$$I_{\rm DBI} = \int_{\mathcal{M}_{p+1}} d^{p+1}\xi \sqrt{-\det\left(\Pi_i^a \Pi_j^b \eta_{ab} - \mathcal{F}_{ij}\right)}\,, \tag{3.1}$$

where $\mathcal{M}_{p+1}$ is the worldvolume of the Dp-brane and Π_i^a is the pullback to the worldvolume of the supercurrent corresponding to translations. The two-form $\mathcal{F}$ is the modified field strength of the NS$\otimes$NS two-form gauge potential $\mathrm{B}^{(2)}$, and defined by $\mathcal{F} = db - \mathrm{B}^{(2)}$, where b is the Born-Infeld $\mathrm{U}(1)$ gauge field on the worldvolume. The DBI action is manifestly supersymmetric.

The WZ action for Dp-branes is defined as

$$I_{\rm WZ} = \int_{\mathcal{M}_{p+1}} e^{\mathcal{F}} \wedge \mathcal{C}\,, \quad \mathcal{C} = \bigoplus_n \mathcal{C}^{(n)}\,, \tag{3.2}$$

where $\mathcal{C}^{(n)}$ is characterized by the pullback to the worldvolume of the n-form R⊗R gauge potential $\mathrm{C}^{(n)}$.[2] The WZ action $I_{\rm WZ}$ is superinvariant. The invariance is however not manifest, since the integrand defined by $\mathcal{L}_{\rm WZ} \equiv [e^{\mathcal{F}} \wedge \mathcal{C}]_{p+1}$ is not superinvariant. The WZ term is said to be *quasi*-superinvariant; superinvariant *up to a total divergence*, $\delta\mathcal{L}_{\rm WZ} = d\mathcal{O}$. We show that the WZ term for Dp-branes can be constructed to be manifestly superinvariant in terms of supersymmetric currents on the corresponding supergroups. To do this, we recall that we have introduced h_{p+2} as $h_{p+2} = d\mathcal{L}_{\rm WZ}$ in the previous section. It follows that

$$h^{(3)} = d\mathcal{C}^{(2)}\,, \tag{3.3}$$
$$h^{(5)} = d\mathcal{C}^{(4)} + d\mathcal{F}\mathcal{C}^{(2)}\,, \tag{3.4}$$
$$h^{(7)} = d\mathcal{C}^{(6)} + d\mathcal{F}\mathcal{C}^{(4)}\,, \tag{3.5}$$
$$h^{(9)} = d\mathcal{C}^{(8)} + d\mathcal{F}\mathcal{C}^{(6)}\,. \tag{3.6}$$

We determine $\mathcal{C}^{(k)}, k = 2, 4, 6$, by solving the above equations recursively in terms of supersymmetric currents on the supergroups manifolds corresponding to spacetime superalgebras for D-strings, D3-branes and D5-branes, respectively.

For NSp-branes, the DBI action is expressed as (omitting the background scalar fields)

$$\hat{I}_{\rm DBI} = \int_{\mathcal{M}_{p+1}} d^{p+1}\xi \sqrt{-\det(\Pi_i^a \Pi_j^b \eta_{ab} - \mathcal{G}_{ij})}\,, \tag{3.7}$$

and is manifestly superinvariant. The $\mathcal{G}$ is defined by $\mathcal{G} = dc - \mathrm{C}^{(2)}$, where $\mathrm{C}^{(2)}$ is the R⊗R two-form gauge potential and c is a worldvolume gauge field. Instead of the DBI action, one can use the kinetic term of the Green-Schwarz action for F-strings, which is manifestly supersymmetric too. We define WZ action as

$$\hat{I}_{\rm WZ} = \int_{\mathcal{M}_{p+1}} e^{\mathcal{G}} \wedge \mathcal{B}\,, \qquad \mathcal{B} = \mathcal{B}^{(0)} - \mathcal{B}^{(2)} + \mathcal{B}^{(4)} - \mathcal{B}^{(6)} + \mathcal{B}^{(8)} - \mathcal{B}^{(10)}\,. \tag{3.8}$$

We have introduced $\hat{h}_{p+2}$ as $\hat{h}_{p+2} = d\hat{\mathcal{L}}_{\rm WZ}$ in the previous section. It follows that

$$\hat{h}^{(3)} = -d\mathcal{B}^{(2)}\,, \tag{3.9}$$
$$\hat{h}^{(5)} = d\mathcal{B}^{(4)} - d\mathcal{G}\mathcal{B}^{(2)}\,, \tag{3.10}$$
$$\hat{h}^{(7)} = -d\mathcal{B}^{(6)} + d\mathcal{G}\mathcal{B}^{(4)}\,, \tag{3.11}$$
$$\hat{h}^{(9)} = d\mathcal{B}^{(8)} - d\mathcal{G}\mathcal{B}^{(6)}\,. \tag{3.12}$$

We show that $\mathcal{B}^{(k)}, k = 2, 4, 6$, can be constructed in terms of supersymmetric currents on the supergroups manifolds corresponding to spacetime superalgebras for F-strings, D3-branes and NS5-branes, respectively.

[2] We do not distinguish spacetime gauge potentials and the pullback to the worldvolume of them, but use the same characters, throughout this paper.

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3.1 D- and F-strings

In order for the present paper to be self-contained, we begin with describing the IIB D- and F-strings following [15, 16].

For D-strings, we introduce LI Maurer-Cartan (MC) equations corresponding to supertranslations:

$$d\Pi^a = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma^a 1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \qquad d\Pi^{\boldsymbol{\alpha}} = 0\,, \tag{3.13}$$

and D-strings:

$$\begin{aligned} d\Pi'_a &= -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \\ d\Pi'_{\boldsymbol{\alpha}} &= -\Pi^{\boldsymbol{\beta}}\Pi'_a(\gamma^a 1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \Pi^{\boldsymbol{\beta}}\Pi^a(\gamma_a\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \end{aligned} \tag{3.14}$$

which are T-dual to the MC equations for D0- and D2-branes obtained from the M-algebra as will be seen in section 4.2. MC equations contain equivalent information about the algebra:

$$[D_A, D_B\} = f_{AB}{}^C D_C \Longleftrightarrow d\Pi^C = -\frac{1}{2}\Pi^B \wedge \Pi^A f_{AB}{}^C\,, \tag{3.15}$$

where D_A is dual to Π^A and denotes the LI generators of the right-transformation collectively. The structure constants of the supersymmetry algebra generated by the RI generators Q_A of the left-transformation is identical to those of the algebra generated by the LI generators D_A of the right-transformation up to a sign. Keeping this in mind, we call the MC equations of the LI one-forms the dual to the superalgebra for short, throughout this paper. The Jacobi identities of the algebra are satisfied if and only if the integrability conditions $d^2 = 0$ on the dual forms hold. We call the superalgebra corresponding to (3.13) and (3.14) the D-string superalgebra. The supersymmetric WZ term $\mathcal{L}_{\rm WZ} = \mathcal{C}^{(2)}$ for D-strings is constructed in terms of these supercurrents on the corresponding supergroup manifold as[3]

$$\mathcal{C}^{(2)} = -\Pi^a\Pi'_a - \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi'_{\boldsymbol{\alpha}}\,. \tag{3.16}$$

In fact, one can see that $d\mathcal{C}^{(2)} = \Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}$, and then $h_3 = h^{(3)}$, which is (2.10) given by the FDA $\mathfrak{F}$, is obtained. This is consistent with (2.25) of the FDA $\mathfrak{G}$.

In turn for F-strings, MC equations for supertranslation (3.13) and F-strings:

$$\begin{aligned} d\Pi_a &= +\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} \\ d\Pi_{\boldsymbol{\alpha}} &= +\Pi^{\boldsymbol{\beta}}\Pi^a(\gamma_a\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \Pi^{\boldsymbol{\beta}}\Pi_a(\gamma^a 1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \end{aligned} \tag{3.17}$$

are needed. We call the corresponding superalgebra the F-string superalgebra. In

[3]This (pullback of the) two-form gauge potential $\mathcal{C}^{(2)}$ is not used for the D-string action, rather is used for (p,q)-string action. We do not distinguish between them throughout this paper.

terms of these supercurrents on the corresponding supergroup manifold, supersymmetric WZ term $\hat{\mathcal{L}}_{\rm WZ} = -\mathcal{B}^{(2)}$ for F-strings is constructed by

$$\mathcal{B}^{(2)} = -\Pi^a\Pi_a - \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi_{\boldsymbol{\alpha}}\,. \tag{3.18}$$

In fact, one can get $d\mathcal{B}^{(2)} = -\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_a\sigma_3)_{\boldsymbol{\alpha\beta}}$, and then $\hat{h}_3 = \hat{h}^{(3)}$, which is (2.28) obtained from the FDA $\mathfrak{G}$. This is consistent with (2.4) of the FDA $\mathfrak{F}$.

New coordinates for D- and F-string charges are contained in a total derivative term. $h^{(3)}$ and $\hat{h}^{(3)}$ can be regarded as the pullback to the worldvolume of the field strengths of the spacetime gauge potentials, $\mathrm{H}^{(3)} = d\mathrm{C}^{(2)}$ and $-\mathrm{G}^{(3)} = -d\mathrm{B}^{(2)}$, which are invariant under the spacetime gauge transformations

$$\delta\mathrm{C}^{(2)} = d\Lambda^{(1)}\,, \qquad \delta\mathrm{B}^{(2)} = d\Omega^{(1)}\,. \tag{3.19}$$

We regard $\mathcal{B}^{(2)}$ and $\mathcal{C}^{(2)}$ as (the pullback to the worldvolume of) the spacetime gauge potentials $\mathrm{B}^{(2)}$ and $\mathrm{C}^{(2)}$, respectively. The S-duality

$$\mathrm{B}^{(2)} \to -\mathrm{C}^{(2)}\,, \qquad \mathrm{C}^{(2)} \to \mathrm{B}^{(2)}\,, \tag{3.20}$$

maps the F-string WZ term $\hat{\mathcal{L}}_{\rm WZ}$ to the D-string one $\mathcal{L}_{\rm WZ}$. The S-duality transformations (3.20) can be expressed as a set of transformations of the supercurrents:

$$\begin{aligned} &\Pi^a \to \Pi^a\,, &&\Pi_a \to -\Pi'_a\,, &&\Pi'_a \to \Pi_a\,,\\ &\Pi^{\boldsymbol{\alpha}} \to \sigma\Pi^{\boldsymbol{\alpha}}\,, &&\Pi_{\boldsymbol{\alpha}} \to -\sigma\Pi'_{\boldsymbol{\alpha}}\,, &&\Pi'_{\boldsymbol{\alpha}} \to \sigma\Pi_{\boldsymbol{\alpha}}\,, \end{aligned} \tag{3.21}$$

where $\sigma = \pm 1/\sqrt{2}(1 + i\sigma_2)$. These transformations are regarded as the generalized R-symmetry which acts not only on $\Pi^{\boldsymbol{\alpha}}$ but also on $\Pi'_{\boldsymbol{\alpha}}$ and $\Pi_{\boldsymbol{\alpha}}$, as will be seen in the section 4.2.

3.2 D3-branes

For D3-branes, we introduce MC equations:

$$\begin{aligned} d\Pi_{abc} &= \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{abc}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,,\\ d\Pi_{ab\boldsymbol{\alpha}} &= \Pi^{\boldsymbol{\beta}}\Pi_{abc}(\gamma^c 1)_{\boldsymbol{\alpha\beta}} - \Pi^{\boldsymbol{\beta}}\Pi^c(\gamma_{abc}i\sigma_2)_{\boldsymbol{\alpha\beta}} +\\ &\quad +2\Pi^{\boldsymbol{\beta}}\Pi_a(\gamma_b\sigma_1)_{\boldsymbol{\alpha\beta}} + 2\Pi^{\boldsymbol{\beta}}\Pi'_a(\gamma_b\sigma_3)_{\boldsymbol{\alpha\beta}}\,,\\ d\Pi_{a\boldsymbol{\alpha\beta}} &= -\frac{1}{2}\Pi_{abc}\Pi^b(\gamma^c 1)_{\boldsymbol{\alpha\beta}} + \frac{1}{4}\Pi_{ab\boldsymbol{\gamma}}\Pi^{\boldsymbol{\gamma}}(\gamma^b 1)_{\boldsymbol{\alpha\beta}} + 2\Pi_{ab\boldsymbol{\alpha}}\Pi^{\boldsymbol{\gamma}}(\gamma^b 1)_{\boldsymbol{\beta\gamma}} -\\ &\quad -\frac{1}{2}\Pi^b\Pi^c(\gamma_{abc}i\sigma_2)_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi^b\Pi'_b(\gamma_a\sigma_3)_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi^b\Pi_b(\gamma_a\sigma_1)_{\boldsymbol{\alpha\beta}} +\\ &\quad +\Pi^b\Pi'_a(\gamma_b\sigma_3)_{\boldsymbol{\alpha\beta}} + \Pi^b\Pi_a(\gamma_b\sigma_1)_{\boldsymbol{\alpha\beta}} - \frac{1}{4}\Pi^{\boldsymbol{\gamma}}\Pi'_{\boldsymbol{\gamma}}(\gamma_a\sigma_3)_{\boldsymbol{\alpha\beta}} -\\ &\quad -\frac{1}{4}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\gamma}}(\gamma_a\sigma_1)_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi_b\Pi'_a\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi_a\Pi'_b\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} -\\ &\quad -2\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha}}(\gamma_a\sigma_1)_{\boldsymbol{\beta\gamma}} - 2\Pi^{\boldsymbol{\gamma}}\Pi'_{\boldsymbol{\alpha}}(\gamma_a\sigma_3)_{\boldsymbol{\beta\gamma}}\,, \end{aligned}$$

$$
\begin{aligned}
d\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} = & \, 2\Pi^a\Pi_{ab\boldsymbol{\alpha}_1}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma^a 1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 5\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^a 1)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} - \\
& -3{\Pi_a}'\Pi_{\boldsymbol{\alpha}_1}(\gamma^a 1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 3\Pi_a\Pi'_{\boldsymbol{\alpha}_1}(\gamma^a 1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - 5\Pi^a\Pi'_{\boldsymbol{\alpha}_1}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - \\
& -5\Pi^a\Pi_{\boldsymbol{\alpha}_1}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,, \qquad (3.22)
\end{aligned}
$$

in addition to MC equations for supertranslations (3.13), F-strings (3.17) and D-strings (3.14). We call the superalgebra corresponding to (3.13), (3.17), (3.14) and (3.22) the D3-brane superalgebra. The MC equations (3.22) are T-dual to those of D2-branes and D4-branes obtained from the M-algebra as will be seen in section 4.3.

We find that the supersymmetric WZ term is constructed as

$$
\mathcal{L}_{\mathrm{WZ}} = \mathcal{C}^{(4)} + \mathcal{F}\wedge\mathcal{C}^{(2)}\,, \qquad (3.23)
$$

where

$$
\begin{aligned}
\mathcal{C}^{(4)} &= \mathrm{C}^{(4)} + \frac{1}{2}\mathrm{C}^{(2)}\mathrm{B}^{(2)}\,, \qquad (3.24)\\
\mathrm{C}^{(4)} &= \frac{1}{6}\Pi^a\Pi^b\Pi^c\Pi_{abc} - \frac{29}{140}\Pi^a\Pi^b\Pi^{\boldsymbol{\alpha}}\Pi_{ab\boldsymbol{\alpha}} - \frac{3}{35}\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
&\quad + \frac{1}{140}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha}\boldsymbol{\beta}\boldsymbol{\gamma}} - \frac{3}{35}\Pi^a\Pi_a\mathrm{C}^{(2)} + \frac{3}{35}\Pi^a{\Pi_a}'\mathrm{B}^{(2)}, \qquad (3.25)
\end{aligned}
$$

and $\mathrm{C}^{(2)}$ and $\mathrm{B}^{(2)}$ are found in (3.16) and (3.18), respectively. In fact, this implies

$$
d\mathcal{C}^{(4)} - d\mathcal{B}^{(2)}\mathcal{C}^{(2)} = \frac{1}{3}\Pi^{a_1}\Pi^{a_2}\Pi^{a_3}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \qquad (3.26)
$$

and the right-hand side is the $h^{(5)}$ found in (2.11), that is $d\mathcal{L}_{\mathrm{WZ}} = h_5$. This supersymmetric WZ term is equivalent to the WZ term obtained in section 2 up to a total derivative. New coordinates are contained in a surface term as was shown in the case of F- and D-strings. To see this, we introduce coordinates x^a and $\theta^{\boldsymbol{\alpha}}$ on the supergroup manifold associated with the generators for the supertranslations P_a and $Q_{\boldsymbol{\alpha}}$, respectively. One finds that for any parametrization of the supergroup manifold the supercurrents Π^a and $\Pi^{\boldsymbol{\alpha}}$ are expressed as $\Pi^a = dx^a + \frac{1}{2}(\bar\theta\gamma^a 1 d\theta)$ and $\Pi^{\boldsymbol{\alpha}} = d\theta^{\boldsymbol{\alpha}}$, because of the fiber bundle structure of our superalgebra with the ordinary superspace characterized by (3.13) being the base space. This implies that the terms containing new coordinates in $\mathcal{L}_{\mathrm{WZ}}$ are eliminated by an exterior derivative d and thus contained in a surface term. This is a universal property of our formulation.

We can construct the WZ term for D3-branes in a different form. The supersymmetric WZ term is

$$
\hat{\mathcal{L}}_{\mathrm{WZ}} = \mathcal{B}^{(4)} - \mathcal{G}\wedge\mathcal{B}^{(2)}, \qquad (3.27)
$$

where

$$
\mathcal{B}^{(4)} = \mathrm{C}^{(4)} - \frac{1}{2}\mathrm{C}^{(2)}\mathrm{B}^{(2)}\,. \qquad (3.28)
$$

In fact, one can show that

$$
d\mathcal{B}^{(4)} + d\mathcal{C}^{(2)}\mathcal{B}^{(2)} = \frac{1}{3}\Pi^{a_1}\Pi^{a_2}\Pi^{a_3}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \qquad (3.29)
$$

and the right-hand side is the $\hat{h}^{(5)}$ found in (2.29). We have shown that $d\hat{\mathcal{L}}_{\rm WZ} = h_5$ and new coordinates are contained in a surface term as was seen above. The superinvariant WZ terms $\mathcal{L}_{\rm WZ}$ and $\hat{\mathcal{L}}_{\rm WZ}$ are equivalent to those obtained in section 2 up to a total derivative, and thus the total actions are κ-invariant.

Now let us comment on the relation between $\mathcal{L}_{\rm WZ}$ and $\hat{\mathcal{L}}_{\rm WZ}$. First, $I = I_{\rm DBI} + I_{\rm WZ}$ is transformed to $\hat{I} = \hat{I}_{\rm DBI} + \hat{I}_{\rm WZ}$ by a vector-vector duality transformation [27, 28], $b \leftrightarrow c$. We add a Lagrange multiplier term $\frac{1}{2}\Lambda^{ij}(\mathcal{F}_{ij} - 2\partial_i b_j + {\rm B}^{(2)}_{ij})$ to I. Solving for b_i implies that $\Lambda^{ij} = -\epsilon^{ijkl}\partial_k c_l$, where c_i is a vector field dual to b_i and turns out to be the worldvolume gauge field associated with the R$\otimes$R two-form ${\rm C}^{(2)}$. One obtains $\hat{I}$ after eliminating $\mathcal{F}_{ij}$ using the equations of motion. The term $\frac{1}{2}{\rm C}^{(2)}{\rm B}^{(2)}$ emerged in (3.24) and (3.28) in our formulation. This is consistent with the spacetime gauge transformations as will be seen below.

Second, the action $\hat{I}$ is mapped to the action I under the S-duality transformations: (3.20) and

$$ {\rm C}^{(4)} \to {\rm C}^{(4)} \,, \tag{3.30} $$

$$ \mathcal{G} \to \mathcal{F} \,, \quad \mathcal{F} \to -\mathcal{G} \,, \quad c \to b \,, \quad b \to -c \,. \tag{3.31} $$

These two imply that I is self-dual by itself under the S-duality and the worldvolume vector-vector duality transformation. The same is true for $\hat{I}$.

Third, $h^{(5)}$ and $\hat{h}^{(5)}$ are identical from the expression in terms of the supercurrents as well as the spacetime gauge potentials. These are regarded as the pullback to the worldvolume of the field strength ${\rm H}^{(5)}$ of the spacetime gauge potential ${\rm C}^{(4)}$

$$ {\rm H}^{(5)} = d{\rm C}^{(4)} - \frac{1}{2}{\rm C}^{(2)}d{\rm B}^{(2)} + \frac{1}{2}{\rm B}^{(2)}d{\rm C}^{(2)} \,. \tag{3.32} $$

The Bianchi identity is $d{\rm H}^{(5)} - {\rm H}^{(3)}{\rm G}^{(3)} = 0$, which corresponds to $dh^{(5)} + h^{(3)}d\mathcal{F} = 0$ of (2.8) or $d\hat{h}^{(5)} + \hat{h}^{(3)}d\mathcal{G} = 0$ of (2.26). ${\rm H}^{(5)}$ is invariant under the spacetime gauge transformations: (3.19) and

$$ \delta{\rm C}^{(4)} = d\Lambda^{(3)} + \frac{1}{2}d\Lambda^{(1)}{\rm B}^{(2)} - \frac{1}{2}d\Omega^{(1)}{\rm C}^{(2)} \,. \tag{3.33} $$

$\mathcal{F}$ and $\mathcal{G}$ are gauge invariant by definition of the gauge transformations of the worldvolume gauge fields b and c: $\delta b = d\omega + \Omega^{(1)}$ and $\delta c = d\lambda + \Lambda^{(1)}$, where ω and λ are worldvolume scalars. The gauge invariances of $I_{\rm WZ}$ and $\hat{I}_{\rm WZ}$ are equivalent to the gauge quasi-invariances of $\mathcal{L}_{\rm WZ}$ and $\hat{\mathcal{L}}_{\rm WZ}$, and then to the gauge invariances, $\delta h^{(5)} = \delta h^{(3)} = 0$ and $\delta\hat{h}^{(5)} = \delta\hat{h}^{(3)} = 0$. These are guaranteed by the gauge invariances of ${\rm H}^{(5)}$, ${\rm H}^{(3)}$ and ${\rm G}^{(3)}$.

Finally, the S-duality transformations (3.30) can be expressed as a set of transformations of supercurrents: (3.21) and

$$ \begin{aligned} &\Pi_{abc} \to \Pi_{abc} \,, && \Pi_{ab\boldsymbol{\alpha}} \to \Pi_{ab(\sigma\boldsymbol{\alpha})} \,, \\ &\Pi_{a\boldsymbol{\alpha\beta}} \to \Pi_{a(\sigma\boldsymbol{\alpha})(\sigma\boldsymbol{\beta})} \,, && \Pi_{\boldsymbol{\alpha\beta\gamma}} \to \Pi_{(\sigma\boldsymbol{\alpha})(\sigma\boldsymbol{\beta})(\sigma\boldsymbol{\gamma})} \,, \end{aligned} \tag{3.34} $$

where $\Pi_{ab(\sigma\boldsymbol{\alpha})} = \sigma\binom{\Pi_{ab\tilde{\alpha}}}{\Pi_{ab\alpha}}$ etc. These transformations are shown to be the generalized R-symmetry, and thus the S-duality is realized as an automorphism of the D3-brane superalgebra, as will be seen in the section 4.3.

3.3 D5- and NS5-branes

For D5-branes, we begin with MC equations listed in appendix A, in addition to those for supertranslations (3.13), F-strings (3.17), D-strings (3.14) and D3-branes (3.22). We call the corresponding superalgebra the D5-brane superalgebra. We find that the supersymmetric WZ terms for D5-branes can be constructed in terms of the supercurrents on the corresponding supergroup manifold as

$$\mathcal{L}_{\rm WZ} = \mathcal{C}^{(6)} + \mathcal{F}\mathcal{C}^{(4)} + \frac{1}{2}\mathcal{F}^2\mathcal{C}^{(2)}\,, \tag{3.35}$$

where

$$\mathcal{C}^{(6)} = \mathrm{C}^{(6)} + \frac{1}{3}\mathrm{B}^{(2)}\mathrm{C}^{(4)} + \frac{1}{6}\mathrm{B}^{(2)}\mathrm{B}^{(2)}\mathrm{C}^{(2)} \tag{3.36}$$

and

$$\begin{aligned}
\mathrm{C}^{(6)} = & -\frac{1}{90}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^D_{a_1\cdots a_5} + \frac{281}{13860}\Pi^{a_1}\cdots\Pi^{a_4}\Pi^{\boldsymbol{\alpha}}\Pi^D_{a_1\cdots a_4\boldsymbol{\alpha}} + \\
& + \frac{52}{3465}\Pi^{a_1}\Pi^{a_2}\Pi^{a_3}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^D_{a_1a_2a_3\boldsymbol{\alpha\beta}} - \frac{47}{13860}\Pi^{a_1}\Pi^{a_2}\Pi^{\boldsymbol{\alpha}_1}\Pi^{\boldsymbol{\alpha}_2}\Pi^{\boldsymbol{\alpha}_3}\Pi^D_{a_1a_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& + \frac{1}{462}\Pi^a\Pi^{\boldsymbol{\alpha}_1}\cdots\Pi^{\boldsymbol{\alpha}_4}\Pi^D_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} + \frac{1}{6930}\Pi^{\boldsymbol{\alpha}_1}\cdots\Pi^{\boldsymbol{\alpha}_5}\Pi^D_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} + \\
& + \Pi^a\Pi_a\left[\frac{19}{1115}\Pi^a\Pi^b\Pi^{\boldsymbol{\alpha}}\Pi_{ab\boldsymbol{\alpha}} + \frac{2}{165}\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha\beta}} - \frac{19}{13860}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha\beta\gamma}}\right] + \\
& + \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi_{\boldsymbol{\alpha}}\left[\frac{52}{3465}\Pi^a\Pi^b\Pi^c\Pi_{abc} - \frac{3}{770}\Pi^a\Pi^b\Pi^{\boldsymbol{\alpha}}\Pi_{ab\boldsymbol{\alpha}} + \right. \\
& \qquad \left. + \frac{4}{1155}\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha\beta}} - \frac{1}{1540}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha\beta\gamma}}\right] + \\
& + \frac{3}{385}\mathrm{B}^{(2)}\left(\mathrm{B}^{(2)}\Pi^a\Pi'_a - \mathrm{C}^{(2)}\Pi^a\Pi_a\right) - \frac{1}{231}\Pi^a\Pi_a\left(\mathrm{B}^{(2)}\Pi^b\Pi'_b - \mathrm{C}^{(2)}\Pi^b\Pi_b\right).
\end{aligned} \tag{3.37}$$

In fact, one finds that

$$d\mathcal{C}^{(6)} + \mathcal{C}^{(4)}d\mathcal{F} = \frac{1}{30}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_5}\sigma_1)_{\boldsymbol{\alpha\beta}}\,, \tag{3.38}$$

so that $d\mathcal{L}_{\rm WZ} = h_7$. The supersymmetric WZ term is thus obtained, and new coordinates are contained in a surface term, due to the fiber bundle structure of our superalgebra.

For NS5-branes, we use the MC equations listed in appendix B in exchange for those for D5-branes. The corresponding superalgebra is called the NS5-brane superalgebra. We find that the supersymmetric WZ term for NS5-branes can be constructed as

$$\hat{\mathcal{L}}_{\rm WZ} = -\mathcal{B}^{(6)} + \mathcal{G}\mathcal{B}^{(4)} - \frac{1}{2}\mathcal{G}^2\mathcal{B}^{(2)}\,, \tag{3.39}$$

where

$$\mathcal{B}^{(6)} = {\rm B}^{(6)} - \frac{1}{3}{\rm C}^{(2)}{\rm C}^{(4)} + \frac{1}{6}{\rm C}^{(2)}{\rm C}^{(2)}{\rm B}^{(2)} \tag{3.40}$$

and

$$\begin{aligned}
{\rm B}^{(6)} = & -\frac{1}{90}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^{NS}_{a_1\cdots a_5} + \frac{281}{13860}\Pi^{a_1}\cdots\Pi^{a_4}\Pi^{\boldsymbol{\alpha}}\Pi^{NS}_{a_1\cdots a_4\boldsymbol{\alpha}} + \\
& + \frac{52}{3465}\Pi^{a_1}\Pi^{a_2}\Pi^{a_3}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a_1a_2a_3\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{47}{13860}\Pi^{a_1}\Pi^{a_2}\Pi^{\boldsymbol{\alpha}_1}\Pi^{\boldsymbol{\alpha}_2}\Pi^{\boldsymbol{\alpha}_3}\Pi^{NS}_{a_1a_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& + \frac{1}{462}\Pi^{a}\Pi^{\boldsymbol{\alpha}_1}\cdots\Pi^{\boldsymbol{\alpha}_4}\Pi^{NS}_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} + \frac{1}{6930}\Pi^{\boldsymbol{\alpha}_1}\cdots\Pi^{\boldsymbol{\alpha}_5}\Pi^{NS}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} - \\
& - \Pi^a\Pi'_a\left[\frac{19}{1155}\Pi^a\Pi^b\Pi^{\boldsymbol{\alpha}}\Pi_{ab\boldsymbol{\alpha}} + \frac{2}{165}\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{19}{13860}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha}\boldsymbol{\beta}\boldsymbol{\gamma}}\right] - \\
& - \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi'_{\boldsymbol{\alpha}}\left[\frac{52}{3465}\Pi^a\Pi^b\Pi^c\Pi_{abc} - \frac{3}{770}\Pi^a\Pi^b\Pi^{\boldsymbol{\alpha}}\Pi_{ab\boldsymbol{\alpha}} + \right. \\
& \qquad\qquad \left. + \frac{4}{1155}\Pi^a\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi_{a\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{1}{1540}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha}\boldsymbol{\beta}\boldsymbol{\gamma}}\right] + \\
& + \frac{3}{385}{\rm C}^{(2)}\Big({\rm C}^{(2)}\Pi^a\Pi_a - {\rm B}^{(2)}\Pi^a\Pi'_a\Big) - \frac{1}{231}\Pi^a\Pi'_a\Big({\rm C}^{(2)}\Pi^b\Pi_b - {\rm B}^{(2)}\Pi^b\Pi'_b\Big)\,.
\end{aligned} \tag{3.41}$$

In fact, one find that

$$-d\mathcal{B}^{(6)} + \mathcal{B}^{(4)}d\mathcal{F} = \frac{1}{30}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_5}\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \tag{3.42}$$

so that $d\hat{\mathcal{L}}_{\rm WZ} = \hat{h}_7$. The supersymmetric WZ term is thus obtained, and new coordinates are contained in a surface term. In summary, we obtained the superinvariant WZ terms $\mathcal{L}_{\rm WZ}$ and $\hat{\mathcal{L}}_{\rm WZ}$ of D5- and NS5-branes respectively. The total actions are identical to those found in section 2 up to a surface term, and κ-invariant.

The relations of the superinvariant WZ terms of NS5- and D5-branes are as follows. $h^{(7)}$ and $\hat{h}^{(7)}$ are regarded as the pullback to the worldvolume of the field strengths ${\rm H}^{(7)}$ and $-{\rm G}^{(7)}$ of the spacetime gauge potentials ${\rm C}^{(6)}$ and ${\rm B}^{(6)}$

$$\begin{aligned}
{\rm H}^{(7)} &= d{\rm C}^{(6)} - \frac{2}{3}{\rm C}^{(4)}{\rm G}^{(3)} + \frac{1}{3}{\rm B}^{(2)}{\rm H}^{(5)}\,, \\
{\rm G}^{(7)} &= d{\rm B}^{(6)} + \frac{2}{3}{\rm C}^{(4)}{\rm H}^{(3)} - \frac{1}{3}{\rm C}^{(2)}{\rm H}^{(5)}\,.
\end{aligned} \tag{3.43}$$

The Bianchi identities are

$$d{\rm H}^{(7)} - {\rm H}^{(5)}{\rm G}^{(3)} = 0\,, \qquad d{\rm G}^{(7)} + {\rm H}^{(5)}{\rm H}^{(3)} = 0\,, \tag{3.44}$$

which correspond to $dh^{(7)} + h^{(5)}d\mathcal{F} = 0$ of (2.8) and $d\hat{h}^{(7)} + \hat{h}^{(5)}d\mathcal{G} = 0$ of (2.26). The field strengths (3.43) are invariant under the spacetime gauge transformations: (3.19), (3.33) and

$$\begin{aligned}
\delta \mathrm{C}^{(6)} &= d\Lambda^{(5)} + \frac{2}{3}d\Lambda^{(3)}\mathrm{B}^{(2)} - \frac{1}{3}d\Omega^{(1)}\mathrm{C}^{(4)} - \\
&\quad -\frac{1}{6}d\Omega^{(1)}\mathrm{B}^{(2)}\mathrm{C}^{(2)} + \frac{1}{6}d\Lambda^{(1)}\mathrm{B}^{(2)}\mathrm{B}^{(2)}\,, \qquad (3.45)\\
\delta \mathrm{B}^{(6)} &= d\Omega^{(5)} - \frac{2}{3}d\Lambda^{(3)}\mathrm{C}^{(2)} + \frac{1}{3}d\Lambda^{(1)}\mathrm{C}^{(4)} - \\
&\quad -\frac{1}{6}d\Lambda^{(1)}\mathrm{B}^{(2)}\mathrm{C}^{(2)} + \frac{1}{6}d\Omega^{(1)}\mathrm{C}^{(2)}\mathrm{C}^{(2)}\,. \qquad (3.46)
\end{aligned}$$

The gauge invariances of I and $\hat{I}$ are assured by $\delta h^{(7)} = 0$ and $\delta\hat{h}^{(7)} = 0$, which are guaranteed by the gauge transformations (3.19), (3.33), (3.45) and (3.46). Thus we have obtained the gauge invariant, κ-invariant and manifestly superinvariant IIB D/NS5-brane action.

The WZ terms $\mathcal{L}_{\mathrm{WZ}}$ for D5-branes and $\hat{\mathcal{L}}_{\mathrm{WZ}}$ for NS5-branes are related by the S-duality: (3.20), (3.30), (3.31) and

$$\mathrm{B}^{(6)} \to -\mathrm{C}^{(6)}\,, \qquad \mathrm{C}^{(6)} \to \mathrm{B}^{(6)}\,. \qquad (3.47)$$

Under these transformations, the DBI actions I_{DBI} and $\hat{I}_{\mathrm{DBI}}$ are interchanged, and thus the obtained actions I and $\hat{I}$ are S-dual to each other. The S-duality transformations are expressed as transformations on the supercurrents as

$$\begin{aligned}
\Pi^{NS}_{a_1\cdots a_5} &\to -\Pi^{D}_{a_1\cdots a_5}\,, & \Pi^{NS}_{a_1\cdots a_4\boldsymbol{\alpha}} &\to -\Pi^{D}_{a_1\cdots a_4(\sigma\boldsymbol{\alpha})}\,,\\
\Pi^{NS}_{a_1\cdots a_3\boldsymbol{\alpha}\boldsymbol{\beta}} &\to -\Pi^{D}_{a_1\cdots a_3(\sigma\boldsymbol{\alpha})(\sigma\boldsymbol{\beta})}\,, & \Pi^{NS}_{a_1a_2\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_3} &\to -\Pi^{D}_{a_1a_2(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_3)}\,,\\
\Pi^{NS}_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} &\to -\Pi^{D}_{a(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_4)}\,, & \Pi^{NS}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} &\to -\Pi^{D}_{a(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_5)}\,,\\
\Pi^{D}_{a_1\cdots a_5} &\to \Pi^{NS}_{a_1\cdots a_5}\,, & \Pi^{D}_{a_1\cdots a_4\boldsymbol{\alpha}} &\to \Pi^{NS}_{a_1\cdots a_4(\sigma\boldsymbol{\alpha})}\,,\\
\Pi^{D}_{a_1\cdots a_3\boldsymbol{\alpha}\boldsymbol{\beta}} &\to \Pi^{NS}_{a_1\cdots a_3(\sigma\boldsymbol{\alpha})(\sigma\boldsymbol{\beta})}\,, & \Pi^{D}_{a_1a_2\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_3} &\to \Pi^{NS}_{a_1a_2(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_3)}\,,\\
\Pi^{D}_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} &\to \Pi^{NS}_{a(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_4)}\,, & \Pi^{D}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} &\to \Pi^{NS}_{a(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_5)}\,. \qquad (3.48)
\end{aligned}$$

These transformations are shown to be the generalized R-symmetry, and thus the S-duality is realized as an automorphism of our superalgebra, as will be seen in the sections 4.4 and 4.5.

3.4 Reduction to $N = (1, 0)$

Our superalgebra contains $N = (1, 0)$ superalgebras as its subalgebra.

The F-string superalgebra corresponding to (3.13) and (3.17) is schematically described as

$$\begin{aligned}
\{Q_\alpha, Q_\alpha\} &\sim P^+\,, & \{Q_{\tilde{\alpha}}, Q_{\tilde{\alpha}}\} &\sim P^-\,,\\
[Q_\alpha, P^+] &\sim Z^\alpha\,, & [Q_{\tilde{\alpha}}, P^-] &\sim Z^{\tilde{\alpha}}\,, \qquad (3.49)
\end{aligned}$$

where $P^{\pm}$ denotes $P_a \pm Z^a$. The quotient algebra with respect to the ideal $\{Q_{\tilde\alpha}, P^-, Z^{\tilde\alpha}\}$ is the $N=1$ superalgebra found in [11]. The WZ term for $N=1$ F-strings is characterized by $h^{(3)} = \Pi^a\Pi^\alpha\Pi^\beta(\gamma_a)_{\alpha\beta}$. One finds that the superinvariant WZ term is expressed as $\mathcal{L}_{\rm WZ} = -\mathcal{B}^{(2)}$ where $\mathcal{B}^{(2)} = -\Pi^\alpha\Pi_\alpha$. This agrees with the one found in [10].

Keeping only $Q_{\hat A}$, $Z^{\hat A}$ and $Z^{\hat A_1\cdots\hat A_5}$, where $\hat A$ collectively denotes a and α, in the NS5-brane superalgebra, we obtain $N=(1,0)$ NS5-brane superalgebra, which is similar to those obtained in [29] (with a trivial scaling of $\Pi_{\alpha_1\cdots\alpha_5}$). The WZ term $\mathcal{L}_{\rm WZ}$ is characterized by $h^{(7)} = \frac{1}{30}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^\alpha\Pi^\beta(\gamma_{a_1\cdots a_5})_{\alpha\beta}$. The superinvariant WZ term is $\mathcal{L}_{\rm WZ} = -\mathcal{B}^{(6)}$ where

$$\begin{aligned}\mathcal{B}^{(6)} = &-\frac{1}{90}\Pi^{a_1}\cdots\Pi^{a_5}\Pi^{NS}_{a_1\cdots a_5} + \frac{281}{13860}\Pi^{a_1}\cdots\Pi^{a_4}\Pi^\alpha\Pi^{NS}_{a_1\cdots a_4\alpha} + \\ &+\frac{52}{3465}\Pi^{a_1}\Pi^{a_2}\Pi^{a_3}\Pi^\alpha\Pi^\beta\Pi^{NS}_{a_1a_2a_3\alpha\beta} - \frac{47}{13860}\Pi^{a_1}\Pi^{a_2}\Pi^{\alpha_1}\Pi^{\alpha_2}\Pi^{\alpha_3}\Pi^{NS}_{a_1a_2\alpha_1\alpha_2\alpha_3} + \\ &+\frac{1}{462}\Pi^a\Pi^{\alpha_1}\cdots\Pi^{\alpha_4}\Pi^{NS}_{a\alpha_1\cdots\alpha_4} + \frac{1}{6930}\Pi^{\alpha_1}\cdots\Pi^{\alpha_5}\Pi^{NS}_{\alpha_1\cdots\alpha_5}\,. \end{aligned} \tag{3.50}$$

The new coordinates in the WZ term are found to be contained in a surface term.

4. The IIB-brane superalgebra

In the previous section, we have introduced a set of superalgebras. Let us derive and discuss these superalgebras with keeping the relations to the M-algebra in this section.

Foe this purpose, we briefly describe the relation of branes in the type-IIB theory to those in the M-theory. Eleven-dimensional M-theory contains 32 Majorana supercharges Q_α. The 528 components of the anti-commutator $\{Q_\alpha, Q_\beta\}$ of the supercharges are decomposed into generators for translations, M2-branes and M5-branes as $\mathbf{528} = \mathbf{11} + \mathbf{55} + \mathbf{462}$:

Translation	P_μ	$\mathbf{11}$,
M2-brane (M9-brane)	$Z^{\mu\nu}$	$\mathbf{55}$,
M5-brane (MKK6-brane)	$Z^{\mu_1\cdots\mu_5}$	$\mathbf{462}$,

where $\mu, \nu, \ldots$ are eleven-dimensional Lorentz indices. In the parentheses, we write the branes corresponding to charges with a time index. One calls these branes M-branes collectively.

The IIA-branes in ten-dimensions are obtained by a dimensional reduction of the M-branes with respect to eleven-th direction, say $\natural$-direction. The 32 components of Majorana supercharges in eleven-dimensions are reduced to 16+16 Majorana-Weyl spinors $(Q_\alpha, Q_{\dot\alpha})$ with the opposite chirality. The IIA-branes are related to the

M-branes as

Translation	P_a	$\mathbf{9}\ (\mathbf{1})$,
D0-brane	$P_\natural$	$\mathbf{1}$,
F-string (NS9-brane)	$Z^{\natural a}$	$\mathbf{9}\ (\mathbf{1})$,
D2-brane (D8-brane)	Z^{ab}	$\mathbf{36}\ (\mathbf{9})$,
D4-brane (D6-brane)	$Z^{\natural a_1 \cdots a_4}$	$\mathbf{126}\ (\mathbf{84})$,
NS5-brane (KK5-brane)	$Z^{a_1 \cdots a_5}$	$\mathbf{126}\ (\mathbf{126})$,

where $a, b, \ldots$ are ten-dimensional Lorentz indices. The generators for translations and D0-branes originate from those for translations in eleven-dimensions, the generators for F-strings and D2-branes from those for M2-branes, and the generators for D4-branes and NS5-branes from those for M5-branes.

The IIB-branes are related to the above IIA-branes under a T-duality transformation. The type-IIB theory is chiral, which has 16+16 Majorana-Weyl supercharges $(Q_\alpha, Q_{\tilde{\alpha}})$ with the same chirality. In order to change the chirality of a half of the IIA supercharges, $Q_{\dot{\alpha}}$ are multiplied by a ten-dimensional γ-matrix $\gamma^\sharp$, where the $\sharp$-direction is a space-like direction with respect to which a T-duality is performed. One finds that the obtained algebra is rewritten in a ten-dimensional covariant form under the following identifications of the IIB-brane charges and the M-brane charges:

Translation	$P_a = P_i \oplus Z^{\natural\sharp}$	$\mathbf{10}$,
F-string (NS9-brane)	$Z^a = Z^{\natural i} \oplus P_\sharp$	$\mathbf{9}\ (\mathbf{1})$,
D-string (D9-brane)	$\Sigma^a = Z^{\sharp i} \oplus P_\natural$	$\mathbf{9}\ (\mathbf{1})$,
D3-brane (NS/D7-brane)	$\Sigma^{abc} = Z^{\natural\sharp ijk} \oplus Z^{ij}$	$\mathbf{84}\ (\mathbf{36})$,
D5-brane	$\Sigma^{m_1 \cdots m_5} \sim Z^{\natural 0 p_1 p_2 p_3} \oplus Z^{\natural p_1 \cdots p_4}$	$\mathbf{126}$,
NS5-brane	$Z^{m_1 \cdots m_5} \sim Z^{\sharp p_1 \cdots p_4} \oplus Z^{0 \sharp p_1 p_2 p_3}$	$\mathbf{126}$,
KK5-brane	$P^{0 m_1 \cdots m_4} \sim Z^{0 p_1 \cdots p_4} \oplus Z^{p_1 \cdots p_5}$	$\mathbf{126}$,

where m runs all of the nine spacelike directions, but p runs except for the $\sharp$-direction. i runs ten spacetime directions except for the $\natural$-direction. The D5-, NS5- and KK5-brane charges are self-dual. The "$\sim$" means the equivalence up to the self-duality relation. It follows that the S-duality in the IIB-branes is rephrased as a modular transformation on T^2 spanned by $(\natural, \sharp)$.

4.1 M-algebra, IIA-brane superalgebra and T-duality

The ordinary superspace, which is the supertranslation group ("superPoincaré"/"Lorentz") generated by $Q_A = (P_a, Q_{\boldsymbol{\alpha}})$ is regarded as a generalization of the spacetime which is the translation group ("Poincaré"/"Lorentz") generated by P_a (Lorentz group is an automorphism of the algebra.) We now extend the ordinary superspace to those spanned by generators of p-branes $Z^{A_1 \cdots A_p}$ as well as Q_A. In order to consider the relations of the IIB-brane superalgebra to the M-algebra, we begin with describing the M-algebra [20], which is an eleven dimensional superalgebra generated by[4]

[4]We omit "superstring" contained in the M-algebra.

$$
\begin{array}{ll}
\text{Supertranslation} & Q_M \Longleftrightarrow \Pi^M\,, \\
\text{M2-brane (M9-brane)} & Z^{MN} \Longleftrightarrow \Pi_{MN}\,, \\
\text{M5-brane (MKK6-brane)} & Z^{M_1\cdots M_5} \Longleftrightarrow \Pi_{M_1\cdots M_5}\,,
\end{array}
$$

where eleven-dimensional Lorentz indices μ and 32-component Majorana spinor indices α are collectively denoted as $M, N, \ldots$, so that $Q_M = (P_\mu, Q_\alpha)$, $Z^{MN} = (Z^{\mu\nu}, Z^{\mu\alpha}, Z^{\alpha\beta})$, etc. The right hand sides of arrows denote the corresponding LI one-form on the supergroup manifold.

The IIA-brane superalgebra is obtained by a dimensional reduction of this M-algebra to ten-dimensions, and generated by

$$
\begin{array}{ll}
\text{Supertranslation} & Q_A \Longleftrightarrow \Pi^A = (\Pi^a, \Pi^\alpha, \Pi^{\dot\alpha})\,, \\
\text{F-string (NS9-brane)} & Z^A \Longleftrightarrow \Pi_A = (\Pi_{\natural a}, \Pi_{\natural\alpha}, \Pi_{\natural\dot\alpha})\,, \\
\text{D0-brane} & \Sigma \Longleftrightarrow \Pi = (\Pi^\natural)\,, \\
\text{D2-brane (D8-brane)} & \Sigma^{AB} \Longleftrightarrow \Pi_{AB} = (\Pi_{ab}, \ldots)\,, \\
\text{D4-brane (D6-brane)} & \Sigma^{A_1\cdots A_4} \Longleftrightarrow \Pi_{A_1\cdots A_4} = (\Pi_{\natural a_1\cdots a_4}, \ldots)\,, \\
\text{NS5-brane (KK5-brane)} & Z^{A_1\cdots A_5} \Longleftrightarrow \Pi_{A_1\cdots A_5} = (\Pi_{a_1\cdots a_5}, \ldots)\,,
\end{array}
$$

where the indices $A, B, \ldots$ collectively denote ten-dimensional Lorentz indices a and 16-component Majorana-Weyl spinor indices α and $\dot\alpha$ with positive and negative chirality, respectively. Branes corresponding to the generators with a time index are listed in parentheses. Dots denote the forms with Lorentz indices exchanged for spinorial indices of the corresponding forms.

In order to obtain the type-IIB theory, we must take a T-dual with respect to a space-like direction $x^\sharp$. As a T-duality transformation (see [7]), we perform a dimensional reduction to nine-dimensions with respect to $x^\sharp$ and change the chirality of dotted spinors by multiplying $\gamma^\sharp$ as: $\Pi_{a\tilde\alpha} = (\gamma^\sharp)^{\dot\beta}{}_{\tilde\alpha}\Pi_{a\dot\beta}$ etc.

One may apparently expect that the IIB-brane superalgebra would be completely obtained by a T-duality from the IIA-brane superalgebra. We find however that the generators of supertranslations, F-strings, D-strings and D3-branes originate from those of the IIA-brane superalgebra, but some charges of IIB D5- and IIB NS5-branes are not obtained by a T-duality as will be seen below.

4.2 Supertranslations, F-strings and D-strings

The superalgebra in presence of F-strings and/or D-strings are related to those in presence of F-strings and/or D0- and D2-branes in the IIA superalgebra under a T-duality. This is found to be rewritten in a ten-dimensional covariant form under the identification [15]:

$$
\begin{array}{llll}
\text{Supertranslation} & Q_A \Longleftrightarrow & \hat\Pi^a = (\Pi^i, \Pi_{\natural\sharp})\,, & \hat\Pi^{\boldsymbol\alpha} = \Pi^{\boldsymbol\alpha}\,, \\
\text{F-string} & Z^A \Longleftrightarrow & \hat\Pi_a = (\Pi_{\natural i}, \Pi^\sharp)\,, & \hat\Pi_{\boldsymbol\alpha} = \Pi_{\natural\boldsymbol\alpha}\,, \\
\text{D-string} & \Sigma^A \Longleftrightarrow & \hat\Pi'_a = (\Pi_{\sharp i}, -\Pi^\natural)\,, & \hat\Pi'_{\boldsymbol\alpha} = \Pi_{\sharp\boldsymbol\alpha}\,.
\end{array}
\tag{4.1}
$$

The F-string superalgebra is generated by (Q_A, Z^A) and the corresponding MC equations are (3.13) and (3.17). The D-string superalgebra is generated by (Q_A, Σ^A) and the corresponding MC equations are (3.13) and (3.14). We denote these superalgebras as $sP[Z^A]$ and $sP[\Sigma^A]$, respectively. The Jacobi identities, or equivalently the integrability conditions $d^2 = 0$, are guaranteed by the Fierz identities: $(\gamma_a)_{(\alpha\beta}(\gamma^a)_{\gamma\delta)} = 0$ for F-strings, and $(\gamma_a)_{\tilde{\alpha}(\beta}(\gamma^a)_{\gamma\delta)} = 0$ for D-strings, which are encountered in (2.32) and (2.17) in the CE cohomology classification of IIB-branes.

The S-duality is expressed as a modular transformation on T^2 spanned by $(\natural, \sharp)$. Now, this generalizes to include new fermionic generators $\hat{\Pi}_{\boldsymbol{\alpha}}$ and $\hat{\Pi}'_{\boldsymbol{\alpha}}$. One finds that under the S-duality transformation (3.21), the spacetime superalgebra (3.17) interchanges with (3.14) and (3.13) stays invariant.

The superalgebra generated by generators listed in (4.1), denoting $sP[Z^A, \Sigma^A]$, contains various subalgebras. Noting that new fermionic generators are the centers of the algebra $sP[Z^A, \Sigma^A]$, and that Z^a and Σ^a are the centers of the algebra $sP[Z^a, \Sigma^a]$, one finds

$$sP\Big[Z^A, \Sigma^A\Big] \supset \left\{ \begin{array}{c} sP\left[\binom{Z^\alpha Z^{\tilde{\alpha}}}{Z^a}, \binom{\Sigma^\alpha\ \ }{\ \ \Sigma^a}\right] \\ \vdots \\ sP\left[\binom{\ \ Z^{\tilde{\alpha}}}{Z^a}, \binom{\Sigma^\alpha \Sigma^{\tilde{\alpha}}}{\Sigma^a}\right] \end{array} \right. \supset \cdots \supset sP[Z^a, \Sigma^a] \supset \left\{ \begin{array}{c} sP[Z^a] \\ sP[\Sigma^a] \end{array} \right. \supset sP\,, \quad (4.2)$$

where sP is generated by Q_A as (3.13). The superalgebra $sP[Z^A, \Sigma^A]$ contains $sP[Z^A]$ or $sP[\Sigma^A]$ as a subalgebra since Z^A and Σ^A are ideals of the algebra $sP[Z^A, \Sigma^A]$

$$sP\left[Z^A, \Sigma^A\right] \supset \left\{ \begin{array}{l} sP[Z^A] \supset \left\{ \begin{array}{l} sP[\binom{Z^\alpha\ \ }{\ \ Z^a}] \\ sP[\binom{\ \ Z^{\tilde{\alpha}}}{Z^a}] \end{array} \right. \supset sP[Z^a]\,, \\ \\ sP[\Sigma^A] \supset \left\{ \begin{array}{l} sP[\binom{\Sigma^\alpha\ \ }{\ \ \Sigma^a}] \\ sP[\binom{\ \ \Sigma^{\tilde{\alpha}}}{\Sigma^a}] \end{array} \right. \supset sP[\Sigma^a]\,, \end{array} \right. \supset sP\,. \quad (4.3)$$

Due to the property that the superalgebras contain sP as a subalgebra, coordinates corresponding to the generators Z^A and Σ^A are not contained in supercurrents Π^A. The fiber bundle structure with the base space sP is a universal structure of our superalgebras.

4.3 D3-branes

The D3-brane charges with three bosonic indices are related to those of the wrapped D4-brane and unwrapped D2-brane in the IIA superalgebra, which are $Z^{p_1p_2}$ and $Z^{\natural\sharp p_1p_2p_3}$ in the M-algebra, where p_i runs 1 to 9 except for $\sharp$. We find that the superalgebra obtained by a T-duality from the IIA D2- and D4-brane superalgebra is rewritten in a ten-dimensional covariant form and the D3-brane charges Σ^{ABC} are completely determined in terms of the charges in the M-algebra:

$$\text{D3-brane } \Sigma^{ABC} \iff \begin{cases} \hat{\Pi}_{abc} = \left(\hat{\Pi}_{ijk} = \Pi_{\natural\sharp ijk}, \hat{\Pi}_{\sharp ij} = -\Pi_{ij}\right), \\ \hat{\Pi}_{ab\boldsymbol{\alpha}} = \left(\hat{\Pi}_{ij\boldsymbol{\alpha}} = \Pi_{\natural\sharp ij\boldsymbol{\alpha}}, \hat{\Pi}_{\sharp i\boldsymbol{\alpha}} = \Pi_{i\boldsymbol{\alpha}}\right), \\ \hat{\Pi}_{a\boldsymbol{\alpha\beta}} = \left(\hat{\Pi}_{i\boldsymbol{\alpha\beta}} = \Pi_{\natural\sharp i\boldsymbol{\alpha\beta}}, \hat{\Pi}_{\sharp\boldsymbol{\alpha\beta}} = -\Pi_{\boldsymbol{\alpha\beta}}\right), \\ \hat{\Pi}_{\boldsymbol{\alpha\beta\gamma}} = \Pi_{\natural\sharp\boldsymbol{\alpha\beta\gamma}}. \end{cases} \tag{4.4}$$

The S-duality, which is a modular transformation on T^2 spanned by $(\natural, \sharp)$, generalizes to include new fermionic generators $\Sigma^{ab\boldsymbol{\alpha}}$, $\Sigma^{a\boldsymbol{\alpha\beta}}$ and $\Sigma^{\boldsymbol{\alpha\beta\gamma}}$. We obtain the D3-brane superalgebra (dropping hats) found in (3.22). Jacobi identities are satisfied due to

$$(\gamma^{ab}{}_{c})_{\tilde{\alpha}(\beta}(\gamma^{c})_{\gamma\delta)} + 2\left(\gamma^{[a}\right)_{\tilde{\alpha}(\beta}\left(\gamma^{b]}\right)_{\gamma\delta)} = 0\,, \tag{4.5}$$

which is the characteristic identity of D3-branes, and encountered in (2.18) in the CE cohomology classification of IIB-branes.

The S-duality was expressed as a set of transformations of supercurrents: (3.21) and (3.34), which is a generalization of the R-symmetry. We find that these transformations leave the MC equations (3.22) for D3-branes invariant. The S-duality associated with the generalized R-symmetry is realized as an automorphism of our algebra.

Interestingly, we find that the existence of the D3-brane charges Z^{ABC} in the superalgebra necessarily requires the presence of the F-string charges Z^A and the D-string charges Σ^A, while the $N = 1$ p-brane superalgebra [13] is generated by supertranslation and p-brane charges. This is a characteristic feature of our superalgebras. This is consistent with the fact that the WZ terms for D3-branes are composed of the spacetime gauge potentials which couple to D3-branes, D-strings and F-strings, or that the gauge field strength of the four-form gauge potential which couples to D3-branes contains the gauge potentials which couple to D-strings and F-strings.

The D3-brane superalgebra contains the F- and D-string superalgebras as subalgebras. In addition, the D3-brane superalgebra contains various subalgebras. We assign a degree $(p; q, \tilde{q})$ to the dual form $\Pi_{a_1\cdots a_p\alpha_1\cdots\alpha_q\tilde{\beta}_1\cdots\tilde{\beta}_{\tilde{q}}}$ of generators. Under an operation of the differential d, the forms with degree $(p; q, \tilde{q})$ transform to those with degree:

$$\begin{aligned} &(p+2; q-2, \tilde{q})\,, \qquad (p+2; q, \tilde{q}-2)\,, \qquad (p+1; q-2, \tilde{q}+1)\,, \\ &(p+1; q-1, \tilde{q})\,, \qquad (p+1; q, \tilde{q}-1)\,, \qquad (p+1; q+1, \tilde{q}-2)\,, \end{aligned} \tag{4.6}$$

and those with degree $(p'; q', \tilde{q}')$ where $p' + q' + \tilde{q}' = 1$. In order to close an algebra, Jacobi identities, or equivalently $d^2 = 0$, must be satisfied. A subalgebra is generated by the forms with degree $(p; q, \tilde{q})$, those with degree (4.6) and those obtained by differentiating the forms with degree (4.6). The new generators $\Sigma^{\boldsymbol{\alpha\beta\gamma}}$ are the centers of the D3-brane superalgebra $sP[Z^A, \Sigma^A, \Sigma^{ABC}]$, and the new generators $\Sigma^{a\boldsymbol{\alpha\beta}}$ are the centers of the subalgebra $sP[Z^A, \Sigma^A, \Sigma^{aAB}]$, and so on. In addition, $Z^{\boldsymbol{\alpha}}$, $\Sigma^{\boldsymbol{\alpha}}$ and Σ^{abc} are center of the algebra $sP[Z^A, \Sigma^A, \Sigma^{abc}]$. In general, letting $\mathfrak{g}$ be a Lie algebra, the sequences $\mathfrak{h}^0, \mathfrak{h}^1, \ldots$ defined by $\mathfrak{h}^0 \equiv \mathfrak{g}, \mathfrak{h}^1 \equiv [\mathfrak{g}, \mathfrak{h}], \mathfrak{h}^2 \equiv [\mathfrak{g}, \mathfrak{h}^1], \ldots, \mathfrak{h}^n \equiv [\mathfrak{g}, \mathfrak{h}^{n-1}], \ldots$, is called the descending central sequence. These satisfy $\mathfrak{h}^0 \supset \mathfrak{h}^1 \supset \mathfrak{h}^2 \supset \cdots \supset \mathfrak{h}^n \supset \cdots$ and $\mathfrak{h}^0, \mathfrak{h}^1, \mathfrak{h}^2, \ldots$ are ideals in $\mathfrak{g}$. In our case, the descending central sequence is $\mathfrak{h}^0 = sP[Z^A, \Sigma^A, \Sigma^{ABC}]$, $\mathfrak{h}^1 = \{P_a, Z^A, \Sigma^A, \Sigma^{ABC}\}$, $\mathfrak{h}^2 = \{Z^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}AB}\}$, $\mathfrak{h}^3 = \{\Sigma^{\boldsymbol{\alpha\beta}A}\}$, $\mathfrak{h}^4 = \{\Sigma^{\boldsymbol{\alpha\beta\gamma}}\}$ and $\mathfrak{h}^5 = \{0\}$. Our algebra is nilpotent and ideals $\mathfrak{h}^1$, $\mathfrak{h}^2$, $\mathfrak{h}^3$ and $\mathfrak{h}^4$ are said to be proper. Especially, $\mathfrak{h}^4$ is abelian ideal, said to be the center in $sP[Z^A, \Sigma^A, \Sigma^{ABC}]$. For a given ideal I, the quotient Lie algebra is defined by $\mathfrak{g}/\sim$, where $\sim$ denotes the equivalent relation: $x \sim x + I$ for $x \in \mathfrak{g}$. The quotient Lie algebra $\mathfrak{g}/\sim$ is a subalgebra of $\mathfrak{g}$. It follows that the D3-brane superalgebra contains various subalgebras:

$$
\begin{aligned}
& sP\left[Z^A, \Sigma^A, \Sigma^{ABC}\right] \supset \begin{cases} sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} \Sigma^{\alpha\beta\gamma}\Sigma^{\alpha\beta\tilde{\gamma}}\Sigma^{\alpha\tilde{\beta}\tilde{\gamma}} & \\ \Sigma^{ABa} & \end{smallmatrix}\right)\right] \\ \quad\vdots \\ sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} & \Sigma^{\alpha\beta\tilde{\gamma}}\Sigma^{\alpha\tilde{\beta}\tilde{\gamma}}\Sigma^{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}} \\ & \Sigma^{ABa} \end{smallmatrix}\right)\right] \end{cases} \supset \cdots \supset \\
& \supset sP\left[Z^A, \Sigma^A, \Sigma^{ABa}\right] \supset \begin{cases} sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} \Sigma^{a\alpha\beta}\Sigma^{a\alpha\tilde{\beta}} & \\ \Sigma^{Aab} & \end{smallmatrix}\right)\right] \\ \quad\vdots \\ sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} & \Sigma^{a\alpha\tilde{\beta}}\Sigma^{a\tilde{\alpha}\tilde{\beta}} \\ & \Sigma^{Aab} \end{smallmatrix}\right)\right] \end{cases} \supset \cdots \supset \\
& \supset sP\left[Z^A, \Sigma^A, \Sigma^{Aab}\right] \supset \begin{cases} sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} \Sigma^{ab\alpha} & \\ \Sigma^{abc} & \end{smallmatrix}\right)\right] \\ sP\left[Z^A, \Sigma^A, \left(\begin{smallmatrix} & \Sigma^{ab\tilde{\alpha}} \\ \Sigma^{abc} & \end{smallmatrix}\right)\right] \end{cases} \\
& \supset sP\left[Z^A, \Sigma^A, \Sigma^{abc}\right] \supset \begin{cases} sP\left[Z^A, \Sigma^A\right] \\ \supset \cdots \supset sP[Z^a, \Sigma^a, \Sigma^{abc}] \end{cases} \supset \cdots \supset sP\,.
\end{aligned} \tag{4.7}
$$

4.4 D5-branes

The D5-brane is related to the unwrapped D4-brane and the wrapped D6-brane under a T-duality. The 126 D5-brane charges with five bosonic charges $\Sigma^{m_1 \cdots m_5}, m_i = 1, \ldots, 9$, are obtained from the 70 D4-brane charges $Z^{\natural p_1 \cdots p_4}$ and the 56 D6-brane charges $Z^{\natural 0 p_1 p_2 p_3}$, as $\Sigma^{\sharp p_1 \cdots p_4} = Z^{\natural p_1 \cdots p_4}$ and $\Sigma^{p_1 \cdots p_5} \sim Z^{\natural 0 p_1 p_2 p_3}$, where "$\sim$" denotes the equivalence modulo the self-duality relation. Thus, the charges naturally related to those in the IIA superalgebra are not $\Sigma^{m_1 \cdots m_5}$ but $\Sigma^{\natural i_1 \cdots i_4}$. Under the identification of the D5-brane charges with the charges in the M-algebra,

$$\text{D5-brane } \Sigma^{\sharp I_1\cdots I_4} \Longleftrightarrow \hat{\Pi}^D_{\sharp I_1\cdots I_4} = -\Pi_{\natural I_1\cdots I_4}\,, \tag{4.8}$$

where I collectively denotes i and $\boldsymbol{\alpha}$, we find that the resultant superalgebra is rewritten in a ten-dimensional covariant form. For example, the MC equation

$$\begin{aligned} d\hat{\Pi}^D_{\sharp i_1 i_2 i_3 \alpha} &= -\Pi^{\tilde{\beta}}\hat{\Pi}^a(\gamma_{a\sharp i_1 i_2 i_3})_{\alpha\tilde{\beta}} - \Pi^\beta \hat{\Pi}^D_{a\sharp i_1 i_2 i_3}(\gamma^a)_{\alpha\beta} - \\ &\quad -4\Pi^{\tilde{\beta}}\hat{\Pi}_{[\sharp}(\gamma_{i_1 i_2 i_3]})_{\alpha\tilde{\beta}} + 4\Pi^\beta \hat{\Pi}_{[\sharp i_1 i_2}(\gamma_{i_3]})_{\alpha\beta} \end{aligned} \tag{4.9}$$

is obtained from the IIA D4-brane superalgebra, where hatted forms are defined in (4.1), (4.4) and (4.8). The corresponding Jacobi identity is found to be satisfied by a $(abcd) = (\sharp i_1 \cdots i_4)$ part of the Fierz identity

$$\left(\phi\gamma_e\phi\right)\left(\psi\gamma^{abcde}\phi\right) - 4\left(\phi\gamma^{[a}\phi\right)\left(\psi\gamma^{bcd]}\phi\right) = 0\,, \tag{4.10}$$

which is appeared in (2.19) in the CE cohomology classification of IIB-branes. It follows from this fact that the obtained superalgebra can be lifted into a ten-dimensional covariant form. In addition, the superalgebra can be expressed in terms of SO(2) doublets of the supercurrents: $\Pi^D_{a_1\cdots a_4\boldsymbol{\alpha}} = \binom{\Pi^D_{a_1\cdots a_4\tilde{\alpha}}}{\Pi^D_{a_1\cdots a_4\alpha}}$ etc. As a result, we obtain the MC equations for the D5-brane charges (A.1)–(A.5) in appendix A.

In order to construct the supersymmetric form of the WZ term for D5-branes, we need the charges with five spinorial indices $\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}$. However, the charges with five spinorial indices obtained by a T-duality from the IIA superalgebra, or equivalently the M-algebra, turn out to be the IIB KK5-brane charges, as will be seen in section 4.6. We here introduce the corresponding generators to the algebra for D5-branes. Let us note that under an operation of a differential d, the degree $(p;q,\tilde{q})$ of the form of the D5-brane superalgebra is transformed into a set of the D5-brane charges with degree (4.6). This is a universal property of our superalgebra: F-string, D-string, D3-brane and D5-brane. The D5-brane superalgebra supplied with $\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}$ turns out to possess this property too. This property helps us to construct the MC equations for the charges with five spinorial indices. We can construct the Maurer-Cartan equation (A.6). We find again that the existence of the D5-brane charges necessarily requires the existence of the charges of superalgebra generated by super-translations F-strings, D-strings and D3-branes. This is consistent with the form of the WZ term for D5-branes, or of the seven-form field strength $\mathrm{H}^{(7)}$ of the six-form R⊗R gauge potential $\mathrm{C}^{(6)}$.

The D5-brane superalgebra contains various subalgebras. The descending central series are $\mathfrak{h}^0 = sP[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}]$, $\mathfrak{h}^1 = \{P_a, Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}\}$, $\mathfrak{h}^2 = \{Z^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}AB}, \Sigma^{\boldsymbol{\alpha}A_1\cdots A_4}\}$, $\mathfrak{h}^3 = \{\Sigma^{\boldsymbol{\alpha\beta}A}, \Sigma^{\boldsymbol{\alpha\beta}A_1A_2A_3}\}$, $\mathfrak{h}^4 = \{\Sigma^{\boldsymbol{\alpha\beta\gamma}}, \Sigma^{\boldsymbol{\alpha\beta\gamma}A_1A_2}\}$, $\mathfrak{h}^5 = \{\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4 A}\}$, $\mathfrak{h}^6 = \{\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}\}$, $\mathfrak{h}^7 = \{0\}$. These are the ideals of our algebra, and $\mathfrak{h}^6$ is the center of $sP[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}]$. The generators $\Sigma^{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}$ are the centers of the subalgebra $sP[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{aA_1\cdots A_4}]$, and so on. It follows from this fact that

$$\begin{aligned} & sP\left[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}\right] \supset \left\{ \begin{array}{c} sP\left[Z^A, \Sigma^A, \Sigma^{ABC}, \left(\begin{smallmatrix}\Sigma^{\alpha_1\cdots\alpha_5}\ldots\Sigma^{\alpha\tilde\beta_1\cdots\tilde\beta_4} \\ \Sigma^{A_1\cdots A_4 a}\end{smallmatrix}\right)\right] \\ \vdots \\ sP\left[Z^A, \Sigma^A, \Sigma^{ABC}\left(\begin{smallmatrix}\Sigma^{\alpha_1\cdots\alpha_4\tilde\beta}\ldots\Sigma^{\tilde\beta_1\cdots\tilde\beta_5} \\ \Sigma^{A_1\cdots A_4 a}\end{smallmatrix}\right)\right] \end{array} \right. \supset\cdots\supset \\ & \supset sP\left[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_4 a}\right] \supset\cdots\supset sP\left[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{a_1\cdots a_5}\right] \supset \\ & \supset \left\{ \begin{array}{l} sP\left[Z^A, \Sigma^A, \Sigma^{ABC}\right] \\ \supset\cdots\supset sP\left[Z^a, \Sigma^a, \Sigma^{abc}, \Sigma^{a_1\cdots a_5}\right] \end{array} \right. \supset\cdots\supset sP\,. \end{aligned} \tag{4.11}$$

4.5 NS5-branes

As is well known, under a T-duality: $R \leftrightarrow l_s^2/R$ and $g_s \leftrightarrow l_s g_s/R$, where g_s and l_s denote the string coupling and the string length, respectively, and R is the radius of the $\sharp$-direction, the tension $R/g_s^2 l_s^6$ of the wrapped IIA NS5-brane is transformed into the tension $R/g_s^2 l_s^6$ of the wrapped IIB NS5-brane. The tension $R^2/g_s^2 l_s^8$ of the IIA KK5-brane, with the R identified with the radius of the S^1 of the Taub-NUT geometry, is transformed into the tension $1/g_s^2 l_s^6$ of the unwrapped IIB NS5-brane. The 126 IIB NS5-brane charges with five bosonic indices $Z^{m_1\cdots m_5}$ are obtained from to the 70 IIA NS5-brane charges $Z^{\sharp p_1\cdots p_4}$ and the 56 IIA KK5-brane charges $Z^{\sharp 0 p_1 p_2 p_3}$, as $Z^{\sharp p_1\cdots p_4} = Z^{\sharp p_1\cdots p_4}$ and $Z^{p_1\cdots p_5} \sim Z^{\sharp 0 p_1 p_2 p_3}$, where "$\sim$" denotes the equivalence modulo the self-duality relation. The charges naturally related to those in the IIA superalgebra are not $Z^{m_1\cdots m_5}$ but $Z^{\sharp i_1\cdots i_4}$. Under the identification of the charges of the M-algebra with those in NS5-brane superalgebra:

$$\text{NS5-brane } Z^{\sharp I_1\cdots I_4} \Longleftrightarrow \hat\Pi^{NS}_{\sharp I_1\cdots I_4} = \Pi_{\sharp I_1\cdots I_4}\,. \tag{4.12}$$

We find that the algebra is lifted into a ten-dimensional covariant form. For example, the equation

$$\begin{aligned} d\hat\Pi^{NS}_{\sharp i_1 i_2 i_3 \alpha} &= \Pi^\beta \hat\Pi^a (\gamma_{a\sharp i_1 i_2 i_3})_{\alpha\beta} + \Pi^\beta \hat\Pi^{NS}_{a\sharp i_1 i_2 i_3} (\gamma^a)_{\alpha\beta} - \\ & \quad -4\Pi^{\tilde\beta} \hat\Pi'_{[\sharp} \left(\gamma_{i_1 i_2 i_3]}\right)_{\alpha\tilde\beta} + 4\Pi^{\tilde\beta} \hat\Pi_{[i_1 i_2} \left(\gamma_{i_3]}\right)_{\alpha\tilde\beta} \end{aligned} \tag{4.13}$$

is obtained by a T-duality, where hatted forms are defined in (4.1), (4.4) and (4.12). The corresponding Jacobi identities are satisfied by $(abcd) = (\sharp i_1 i_2 i_3)$ part of the Fierz identity in ten-dimensions

$$(\phi\gamma_e\phi)(\psi\gamma^{abcde}\psi) - (\psi\gamma_e\psi)(\phi\gamma^{abcde}\phi) + 16\left(\phi\gamma^{[a}\right)\left(\phi\gamma^{bcd]}\psi\right) = 0\,, \tag{4.14}$$

which is (2.34) in the CE cohomology classification of IIB-branes. It follows from this fact that the superalgebra can be lifted into a ten-dimensional covariant form. The resulting MC equations are found to be expressed in terms of SO(2) doublets of the supercurrents: $\Pi^{NS}_{a_1\cdots a_4\boldsymbol{\alpha}} = \left(\begin{smallmatrix}\Pi^{NS}_{a_1\cdots a_4\tilde\alpha} \\ \Pi^{NS}_{a_1\cdots a_4\alpha}\end{smallmatrix}\right)$ as the NS5-brane superalgebra listed in (B.1)–(B.5) in appendix B.

In order to construct the supersymmetric WZ term for NS5-branes, we supply the superalgebra with the charges with five spinorial indices $Z^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}$. Noting that the MC equations for NS5-branes (B.1)–(B.5) possess the property that the charges with degree $(p;q,\tilde{q})$ transform into those with degree (4.6), we find the MC equation (B.6) in appendix B.

We find that the existence of the NS5-brane charges in the superalgebra necessarily requires the presence of the charges of F-strings, D-strings and D3-branes. This is consistent with the form of the WZ term for NS5-branes, or of the seven-form field strength $\mathrm{G}^{(7)}$ of the six-form NS⊗NS gauge potential $\mathrm{B}^{(6)}$. The NS5-brane superalgebra contains various subalgebras, which is expressed as (4.11) with replacing the D5-brane charges $\Sigma^{A_1\cdots A_5}$ with the NS5-brane charges $Z^{A_1\cdots A_5}$.

The S-duality was given by a set of transformations of supercurrents (3.21), (3.34) and (3.48). We can show that under these transformations the NS5-brane superalgebra is mapped to the D5-brane superalgebra, and thus the S-duality is realized as an automorphism of $sP[Z^A, \Sigma^A, \Sigma^{ABC}, Z^{A_1\cdots A_5}, \Sigma^{A_1\cdots A_5}]$.

4.6 KK5-branes

Under a T-duality, the tension $1/g_s^2 l_s^6$ of the unwrapped IIA NS5-brane is transformed into the tension $R^2/g_s^2 l_s^8$ of the IIB KK5-brane with the R identified with the radius R_{TN} of the S^1 in the Taub-NUT geometry. The tension $RR_{TN}^2/g_s^2 l_s^8$ of the wrapped IIA KK5-brane is transformed into the tension $RR_{TN}^2/g_s^2 l_s^8$ of the wrapped IIB KK5-brane. The 126 IIB KK5-brane charges with five bosonic indices $Z^{0m_1\cdots m_4}$ are related to the 56 IIA NS5-brane charges $Z^{p_1\cdots p_5}$ and the 70 IIA KK5-brane charges $Z^{0p_1\cdots p_4}$ as $Z^{0\sharp p_1 p_2 p_3} \sim Z^{p_1\cdots p_5}$ and $Z^{0p_1\cdots p_4} = Z^{0p_1\cdots p_4}$, where "$\sim$" denotes the equivalence modulo the self-duality relation. The charges naturally related to those in the IIA superalgebra are not $Z^{0m_1\cdots m_4}$ but $Z^{i_1\cdots i_5}$. As was mentioned before, the algebra obtained by a T-duality is not rewritten in a covariant form. The MC equations for the KK5-brane charges are found in appendix C. The identification of the IIB KK5-brane charges with the M5-brane charges are

$$\text{KK5-brane } P^{I_1\cdots I_5} \Longleftrightarrow \Pi^{KK}_{I_1\cdots I_5} = \Pi_{I_1\cdots I_5}\,. \tag{4.15}$$

The characteristic identity[5] for the KK5-branes is found to be

$$\begin{aligned}(\gamma^i 1)_{\boldsymbol{\alpha\beta}}(\gamma_{ji_1\cdots i_4}1)_{\boldsymbol{\gamma\delta}} + (\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha\beta}}(\gamma_{\sharp i_1\cdots i_4}\sigma_3)_{\boldsymbol{\gamma\delta}} + (\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha\beta}}(\gamma_{\sharp i_1\cdots i_4}\sigma_1)_{\boldsymbol{\gamma\delta}}\\ -3(\gamma_{\sharp i_1 i_2} i\sigma_2)_{\boldsymbol{\alpha\beta}}(\gamma_{\sharp i_3 i_4} i\sigma_2)_{\boldsymbol{\gamma\delta}} = 0\,.\end{aligned} \tag{4.16}$$

[5]Summing up with respect to $\sharp = 0,\ldots,9$ except for $i_1,\ldots,i_4$, the covariant identity is obtained as

$$(\phi\gamma_e\phi)(\psi\gamma^{abcde}\psi) + (\psi\gamma_e\psi)(\phi\gamma^{abcde}\phi) + (\phi\gamma_e\psi)(\psi\gamma^{abcde}\phi) + 3(\phi\gamma^{e[ab}\psi)(\psi\gamma_e{}^{cd]}\phi) = 0\,.$$

Under the S-duality transformation of supercurrents: (3.21), (3.34), (3.48) and

$$
\begin{aligned}
\Pi^{KK}_{i_1\cdots i_5} &\to \Pi^{KK}_{i_1\cdots i_5}\,, & \Pi^{KK}_{i_1\cdots i_4\boldsymbol{\alpha}} &\to \Pi^{KK}_{i_1\cdots i_4(\sigma\boldsymbol{\alpha})}\,,\\
\Pi^{KK}_{i_1 i_2 i_3\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2} &\to \Pi^{KK}_{i_1 i_2 i_3(\sigma\boldsymbol{\alpha}_1)(\sigma\boldsymbol{\alpha}_2)}\,, & \Pi^{KK}_{i_1 i_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} &\to \Pi^{KK}_{i_1 i_2(\sigma\boldsymbol{\alpha}_1)(\sigma\boldsymbol{\alpha}_2)(\sigma\boldsymbol{\alpha}_3)}\,,\\
\Pi^{KK}_{i\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} &\to \Pi^{KK}_{i(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_4)}\,, & \Pi^{KK}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} &\to \Pi^{KK}_{(\sigma\boldsymbol{\alpha}_1)\cdots(\sigma\boldsymbol{\alpha}_5)}\,,
\end{aligned}
\tag{4.17}
$$

the MC equations (C.1)–(C.6) for the KK5-brane charges stay invariant. We find that the existence of the KK5-brane charges in the algebra necessarily requires the presence of the charges of D-strings, F-strings, D3-branes, D5-branes and NS5-branes. The descending central series are $\mathfrak{h}^0 = sP[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}, Z^{A_1\cdots A_5}, P^{I_1\cdots I_5}]$, $\mathfrak{h}^1 = \{P_a, Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}, Z^{A_1\cdots A_5}, P^{I_1\cdots I_5}\}$, $\mathfrak{h}^2 = \{Z^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}}, \Sigma^{\boldsymbol{\alpha}AB}, \Sigma^{\boldsymbol{\alpha}A_1\cdots A_4}, Z^{\boldsymbol{\alpha}A_1\cdots A_4}, P^{\boldsymbol{\alpha}I_1\cdots I_4}\}$, $\mathfrak{h}^3 = \{\Sigma^{\boldsymbol{\alpha\beta}A}, \Sigma^{\boldsymbol{\alpha\beta}A_1A_2A_3}, Z^{\boldsymbol{\alpha\beta}A_1A_2A_3}, P^{\boldsymbol{\alpha\beta}I_1I_2I_3}\}$, $\mathfrak{h}^4 = \{\Sigma^{\boldsymbol{\alpha\beta\gamma}}, \Sigma^{\boldsymbol{\alpha\beta\gamma}AB}, Z^{\boldsymbol{\alpha\beta\gamma}AB}, P^{\boldsymbol{\alpha\beta\gamma}IJ}\}$, $\mathfrak{h}^5 = \{\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4A}, Z^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4A}, P^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4I}\}$, $\mathfrak{h}^6 = \{\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}, Z^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}, P^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}\}$, $\mathfrak{h}^7 = \{0\}$. These are the ideals of our algebra, and $\mathfrak{h}^6$ is the center of the KK5-brane superalgebra $sP[Z^A, \Sigma^A, \Sigma^{ABC}, \Sigma^{A_1\cdots A_5}, Z^{A_1\cdots A_5}, P^{I_1\cdots I_5}]$. Considering quotient algebras, one obtains various subalgebras.

In summary, we have obtained the IIB-brane superalgebra. The S-duality is realized as an automorphism of our algebra. The generators of the M-algebra are mapped to those of the IIB-brane superalgebra, except for the D5-brane charge $\Sigma^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}$ and the NS5-brane charge $Z^{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}$.

5. A description in twelve-dimensions

In the previous section, we obtained the IIB-brane superalgebra in presence of all of the F-strings, D-strings, D3-branes, D5-branes, NS5-branes and KK5-branes. The S-duality is found to be realized as an automorphism of our superalgebra which was the generalized SO(2) R-symmetry. In this section we show this SO(2) symmetry is geometrically realized in twelve-dimensions with signature (11,1).

As the twelve-dimensional Γ-matrices, we define the real representation of $\mathrm{Spin}(11,1)$ using ten-dimensional γ-matrices as:

$$
\Gamma^a = \gamma^a \otimes \sigma_3\,, \qquad \Gamma^\natural = \gamma^\natural \otimes \sigma_3\,, \qquad \Gamma^\flat = 1 \otimes \sigma_1\,. \tag{5.1}
$$

The charge conjugation matrix C in twelve-dimensions is related to one in ten-dimensions c as $C = c \otimes \sigma_3$. It follows that

$$
\begin{aligned}
(\gamma^a 1)_{\boldsymbol{\alpha\beta}} &= (\Gamma^a)_{\boldsymbol{\alpha\beta}}\,, & (\gamma^a \sigma_3)_{\boldsymbol{\alpha\beta}} &= (\Gamma^{\natural a})_{\boldsymbol{\alpha\beta}}\,,\\
(\gamma^a \sigma_1)_{\boldsymbol{\alpha\beta}} &= (\Gamma^{\flat a})_{\boldsymbol{\alpha\beta}}\,, & (\gamma_{abc} i\sigma_2)_{\boldsymbol{\alpha\beta}} &= (\Gamma_{\natural\flat abc})_{\boldsymbol{\alpha\beta}}\\
(\gamma_{a_1\cdots a_5} 1)_{\boldsymbol{\alpha\beta}} &= (\Gamma_{a_1\cdots a_5})_{\boldsymbol{\alpha\beta}}\,, & (\gamma_{a_1\cdots a_5} \sigma_1)_{\boldsymbol{\alpha\beta}} &= (\Gamma_{\flat a_1\cdots a_5})_{\boldsymbol{\alpha\beta}}\,,\\
& & (\gamma_{a_1\cdots a_5} \sigma_3)_{\boldsymbol{\alpha\beta}} &= (\Gamma_{\natural a_1\cdots a_5})_{\boldsymbol{\alpha\beta}}\,.
\end{aligned}
\tag{5.2}
$$

In $(11+1)$-dimensions, p-branes with $p = 1, 2 \bmod 4$ can exist since $(\Gamma^{\mu_1\cdots\mu_p})_{\boldsymbol{\alpha\beta}}$ is symmetric with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ when $p = 1, 2 \bmod 4$. Writing the p-form charges

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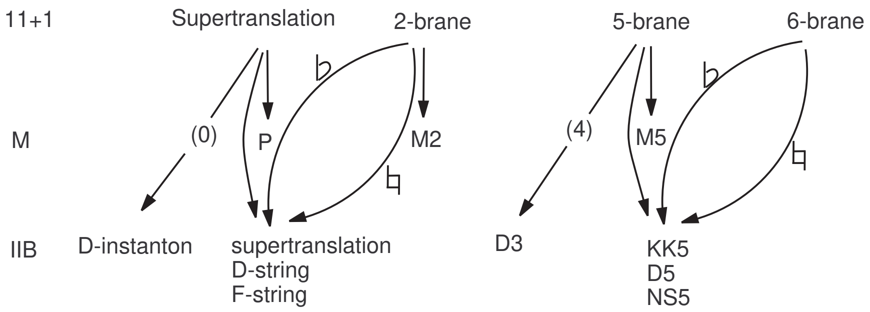

Figure 1: A unification of the IIB-branes and the M-branes in twelve-dimensions.

in twelve-dimensions as $\mathbf{Z}_{\mu_1\cdots\mu_p}$, we find that the IIB-brane charges are expressed in terms of $\mathbf{Z}_{\mu_1\cdots\mu_p}$ as

$$
\begin{aligned}
\mathbf{Z}_A &\Longleftrightarrow \hat{\Pi}^A = \Pi^A\,, && \text{Supertranslation}\,,\\
\mathbf{Z}^{\natural A} &\Longleftrightarrow \hat{\Pi}_{\natural A} = -\Pi_A\,, && \text{F-string}\,,\\
\mathbf{Z}^{\flat A} &\Longleftrightarrow \hat{\Pi}_{\flat A} = \Pi'_A\,, && \text{D-string}\,,\\
\mathbf{Z}^{\natural\flat ABC} &\Longleftrightarrow \hat{\Pi}_{\natural\flat ABC} = \Pi_{ABC}\,, && \text{D3-brane}\,,\\
\mathbf{Z}^{A_1\cdots A_5} &\Longleftrightarrow \hat{\Pi}_{A_1\cdots A_5} = \Pi^{KK}_{A_1\cdots A_5}\,, && \text{KK5-brane}\,,\\
\mathbf{Z}^{\natural A_1\cdots A_5}, &\Longleftrightarrow \hat{\Pi}_{\natural A_1\cdots A_5} = -\Pi^{NS}_{A_1\cdots A_5}\,, && \text{NS5-brane}\,,\\
\mathbf{Z}^{\flat A_1\cdots A_5} &\Longleftrightarrow \hat{\Pi}_{\flat A_1\cdots A_5} = \Pi^{D}_{A_1\cdots A_5}\,, && \text{D5-brane}\,,
\end{aligned}
\tag{5.3}
$$

where the forms with a caret correspond to the dual forms to the charges of the branes in twelve-dimensions. These charge assignments imply that the IIB-branes are related to the branes in twelve-dimensions in such a way that the S-duality is manifestly realized. The relations are depicted in figure 1.

In fact, we can rewrite the MC equations of the IIB-branes as those of the branes in twelve-dimensions. The MC equations for supertranslations, F-strings and D-strings are found to be rewritten as (dropping hats)

$$d\Pi^a = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\Gamma^a)_{\boldsymbol{\alpha\beta}}\,, \tag{5.4}$$

$$d\Pi_{ma} = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\Gamma_{ma})_{\boldsymbol{\alpha\beta}}\,, \tag{5.5}$$

$$d\Pi_{m\boldsymbol{\alpha}} = -\Pi^{\boldsymbol{\beta}}\Pi_{ma}(\Gamma^a)_{\boldsymbol{\alpha\beta}} - \Pi^{\boldsymbol{\beta}}\Pi^a(\Gamma_{ma})_{\boldsymbol{\alpha\beta}}\,, \tag{5.6}$$

where $m = \flat, \natural$. The F- and D-strings are unified as the $\natural$- and $\flat$-wrapped 2-branes in twelve-dimensions, respectively. Projecting the Majorana spinors in twelve-dimensions to 16+16 Majorana-Weyl spinors with the same chirality in ten-dimensions, the above MC equations reduce to those of supertranslations, F-strings and D-strings. The D3-branes are described as the $(\natural, \flat)$-wrapped 5-branes. The MC equations are found to be rewritten as

$$
d\Pi_{m_1 m_2 abc} = \frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\Gamma_{m_1 m_2 abc})_{\boldsymbol{\alpha\beta}}\,, \tag{5.7}
$$

$$
\begin{aligned}
d\Pi_{m_1 m_2 ab\boldsymbol{\alpha}} = {} & \Pi^{\boldsymbol{\beta}}\Pi_{m_1 m_2 abc}(\Gamma^c)_{\boldsymbol{\alpha\beta}} - \Pi^{\boldsymbol{\beta}}\Pi^c(\Gamma_{m_1 m_2 abc})_{\boldsymbol{\alpha\beta}} + \\
& + 6\Pi^{\boldsymbol{\beta}}\Pi_{m_1 m_2}(\Gamma_{ab})_{\boldsymbol{\alpha\beta}}\,,
\end{aligned} \tag{5.8}
$$

$$
\begin{aligned}
d\Pi_{m_1 m_2 a\boldsymbol{\alpha\beta}} = {} & -\frac{1}{2}\Pi_{m_1 m_2 abc}\Pi^b(\Gamma^c)_{\boldsymbol{\alpha\beta}} + \frac{1}{4}\Pi_{m_1 m_2 ab\boldsymbol{\gamma}}\Pi^{\gamma}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + \\
& + 2\Pi_{m_1 m_2 ab\boldsymbol{\alpha}}\Pi^{\gamma}\left(\Gamma^b\right)_{\boldsymbol{\beta\gamma}} - \frac{1}{2}\Pi^b\Pi^c(\Gamma_{m_1 m_2 abc})_{\boldsymbol{\alpha\beta}} + \\
& + \frac{3}{2}\Pi^b\Pi_{m_1 b}(\Gamma_{m_2 a})_{\boldsymbol{\alpha\beta}} + 3\Pi^b\Pi_{m_1 m_2}(\Gamma_{ab})_{\boldsymbol{\alpha\beta}} + \\
& + \frac{3}{4}\Pi^{\gamma}\Pi_{m_1\boldsymbol{\gamma}}(\Gamma_{m_2 a})_{\boldsymbol{\alpha\beta}} - \frac{3}{2}\Pi_{m_1 m_2}\Pi_{ab}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + \\
& + 6\Pi^{\gamma}\Pi_{m_1\boldsymbol{\alpha}}(\Gamma_{m_2 a})_{\boldsymbol{\beta\gamma}}\,,
\end{aligned} \tag{5.9}
$$

$$
\begin{aligned}
d\Pi_{m_1 m_2 \boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} = {} & 2\Pi^a\Pi_{m_1 m_2 ab\boldsymbol{\alpha}}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_2} + \Pi^{\boldsymbol{\beta}}\Pi_{m_1 m_2 a\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\Gamma^a)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& + 5\Pi^{\boldsymbol{\beta}}\Pi_{m_1 m_2 a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^a)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} - 6\Pi_{m_1 a}\Pi_{m_2\boldsymbol{\alpha}_1}(\Gamma^a)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& + 10\Pi^a\Pi_{m_1\boldsymbol{\alpha}_1}(\Gamma_{m_2 a})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,,
\end{aligned} \tag{5.10}
$$

where it is understood that the symmetries of indices on the left-hand side are to be implemented on the right-hand side with unit weight (anti) symmetrizations. Projecting the Majorana spinors in twelve-dimensions to the Majorana-Weyl spinors with the same chirality in ten-dimensions, the above MC equations reduce to those of D3-branes. The IIB KK5-brane are expected to be described as the 5-branes in twelve-dimensions. The MC equations (C.1)–(C.6) for KK5-branes are rewritten in such a way that the S-duality is geometrically realized. The MC equation (C.1) is rewritten as $d\Pi_{i_1\cdots i_5} = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\Gamma_{i_1\cdots i_5})_{\boldsymbol{\alpha\beta}}$, which is similar to (5.7). We expect that the MC equations for 5-branes in twelve-dimensions are composed of (5.7)–(5.10) and the MC equations, which is obtained by lifting the MC equations for KK5-branes into twelve-dimensions.

For NS5- and D5-branes, the corresponding MC equations are found to be rewritten as

$$
d\Pi_{ma_1\cdots a_5} = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\Gamma_{ma_1\cdots a_5})_{\boldsymbol{\alpha\beta}}\,, \tag{5.11}
$$

$$
\begin{aligned}
d\Pi_{ma_1\cdots a_4\boldsymbol{\alpha}} = {} & \Pi^{\boldsymbol{\beta}}\Pi_{ma_1\cdots a_4 b}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + 5\Pi^{\boldsymbol{\beta}}\Pi_{nma_1a_2a_3}({\Gamma^n}_{a_4})_{\boldsymbol{\alpha\beta}} + \\
& + \Pi^{\boldsymbol{\beta}}\Pi^b(\Gamma_{ma_1\cdots a_4 b})_{\boldsymbol{\alpha\beta}} - 5\Pi^{\boldsymbol{\beta}}\Pi_{nm}({\Gamma^n}_{a_1\cdots a_4})_{\boldsymbol{\alpha\beta}}\,,
\end{aligned} \tag{5.12}
$$

$$
\begin{aligned}
d\Pi_{ma_1a_2a_3\boldsymbol{\alpha\beta}} = & -\frac{1}{2}\Pi_{ma_1a_2a_3bc}\Pi^b(\Gamma^c)_{\boldsymbol{\alpha\beta}} + \frac{1}{4}\Pi_{ma_1a_2a_3b\boldsymbol{\gamma}}\Pi^{\boldsymbol{\gamma}}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + \\
& +2\Pi_{ma_1a_2a_3b\boldsymbol{\alpha}}\Pi^{\boldsymbol{\gamma}}\left(\Gamma^b\right)_{\boldsymbol{\beta\gamma}} + 2\Pi^b\Pi_{nbma_1a_2}(\Gamma^n{}_{a_3})_{\boldsymbol{\alpha\beta}} + \\
& +\Pi^b\Pi_{nma_1a_2a_3}(\Gamma^n{}_b)_{\boldsymbol{\alpha\beta}} + 2\Pi_{n_1bma_1a_2}\Pi_{n_2a_3}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + \\
& +\frac{1}{2}\Pi_{n_1b}\Pi_{n_2ma_1a_2a_3}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha\beta}} + \Pi^{\boldsymbol{\gamma}}\Pi_{nma_1a_2\boldsymbol{\gamma}}(\Gamma^n{}_{a_3})_{\boldsymbol{\alpha\beta}} + \\
& +8\Pi^{\boldsymbol{\gamma}}\Pi_{nma_1a_2\boldsymbol{\alpha}}(\Gamma^n{}_{a_3})_{\boldsymbol{\beta\gamma}} - 6\Pi_{nm}\Pi_{a_2a_3}(\Gamma^n{}_{a_3})_{\boldsymbol{\alpha\beta}} - \\
& -2\Pi^{\boldsymbol{\gamma}}\Pi_{n\boldsymbol{\alpha}}(\Gamma^n{}_{ma_1a_2a_3})_{\boldsymbol{\beta\gamma}} + \frac{1}{2}\Pi^b\Pi^c(\Gamma_{ma_1a_2a_3bc})_{\boldsymbol{\alpha\beta}} - \\
& -6\Pi_{n_1m}\Pi_{n_2a_1}\delta^{n_1n_2}(\Gamma_{a_2a_3})_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi^b\Pi_{nb}(\Gamma^n{}_{ma_1a_2a_3})_{\boldsymbol{\alpha\beta}} - \\
& -4\Pi^b\Pi_{nm}(\Gamma^n{}_{a_1a_2a_3b})_{\boldsymbol{\alpha\beta}} - \frac{1}{4}\Pi^{\boldsymbol{\gamma}}\Pi_{n\boldsymbol{\gamma}}(\Gamma^n{}_{ma_1a_2a_3})_{\boldsymbol{\alpha\beta}}\,, \qquad (5.13) \\
d\Pi_{ma_1a_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} = & \; 2\Pi^b\Pi_{ma_1a_2bc\boldsymbol{\alpha}_1}(\Gamma^c)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \Pi^{\boldsymbol{\beta}}\Pi_{ma_1a_2b\boldsymbol{\beta\alpha}_1}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +5\Pi^{\boldsymbol{\beta}}\Pi_{ma_1a_2b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} + 6\Pi_{n_1m}\Pi_{n_2a_1a_2b\boldsymbol{\alpha}_1}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +3\Pi_{n_1b}\Pi_{n_2ma_1a_2\boldsymbol{\alpha}_1}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - 6\Pi^b\Pi_{nbma_1\boldsymbol{\alpha}_1}(\Gamma^n{}_{a_2})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +5\Pi^b\Pi_{nma_1a_2\boldsymbol{\alpha}_1}(\Gamma^n{}_b)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 3\Pi^{\boldsymbol{\beta}}\Pi_{nma_1\boldsymbol{\beta\alpha}_1}(\Gamma^n{}_{a_2})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +15\Pi^{\boldsymbol{\beta}}\Pi_{nma_1\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^n{}_{a_2})_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} + 3\Pi_{n_1ma_1a_2b}\Pi_{n_2\boldsymbol{\alpha}_1}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +9\Pi_{ma_1}\Pi_{n\boldsymbol{\alpha}_1}(\Gamma^n{}_{a_2})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - 12\Pi_{m\boldsymbol{\alpha}_1}\Pi_{na_1}(\Gamma^n{}_{a_2})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\
& +15\Pi_{n_1m}\Pi_{n_2\boldsymbol{\alpha}_1}\delta^{n_1n_2}(\Gamma_{a_1a_2})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - 5\Pi^b\Pi_{n\boldsymbol{\alpha}_1}(\Gamma^n{}_{ma_1a_2b})_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,, \qquad (5.14) \\
d\Pi_{m a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} = & -\Pi^b\Pi_{mabc\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^c)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi_{mab\boldsymbol{\beta\alpha}_1\boldsymbol{\alpha}_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi_{mab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} + 2\Pi_{n_1m}\Pi_{n_2ab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -2\Pi_{n_1b}\Pi_{n_2ma\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + 2\Pi^b\Pi_{nmb\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^n{}_a)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -3\Pi^b\Pi_{nma\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^n{}_b)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \frac{3}{5}\Pi^{\boldsymbol{\beta}}\Pi_{nm\boldsymbol{\beta\alpha}_1\boldsymbol{\alpha}_2}(\Gamma^n{}_a)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{12}{5}\Pi^{\boldsymbol{\beta}}\Pi_{nm\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\Gamma^n{}_a)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} + 3\Pi_{n_1mab\boldsymbol{\alpha}_1}\Pi_{n_2\boldsymbol{\alpha}_2}\delta^{n_1n_2}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -6\Pi_{m\boldsymbol{\alpha}_1}\Pi_{n\boldsymbol{\alpha}_2}(\Gamma^n{}_a)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + 3\Pi_{n_1\boldsymbol{\alpha}_1}\Pi_{n_2\boldsymbol{\alpha}_2}\delta^{n_1n_2}(\Gamma_{ma})_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}\,, \qquad (5.15) \\
d\Pi_{m\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} = & -\frac{35}{6}\Pi^{\boldsymbol{\beta}}\Pi_{ma\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\Gamma^a)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}} - \frac{5}{3}\Pi^{\boldsymbol{\beta}}\Pi_{ma\boldsymbol{\beta\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\Gamma^a)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\Pi^a\Pi_{mab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\Gamma^b\right)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \frac{7}{2}\Pi^a\Pi_{nm\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\Gamma^n{}_a)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\frac{5}{2}\Pi_{n_1a}\Pi_{n_2m\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\delta^{n_1n_2}(\Gamma^a)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \\
& -\frac{15}{2}\Pi_{n_1\boldsymbol{\alpha}_1}\Pi_{n_2ma\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\delta^{n_1n_2}(\Gamma^a)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}\,. \qquad (5.16)
\end{aligned}
$$

The NS5- and D5-branes are unified as the $\flat$- and $\natural$-wrapped 6-branes in twelve-dimensions, respectively. In this way, we find (non-covariant) twelve-dimensional description of the IIB-branes.

The superalgebra is essentially ten-dimensional. In order to unify the M-algebra and the IIB-brane superalgebra, one must lift the superalgebra to a superalgebra with the eleven-dimensional covariance. The relation of the branes in twelve-dimensions to the M-branes are described in figure 1. The eleven-dimensional supertranslations are described in terms of $\mathbf{Z}_M$. The 2- and 5-brane charges $\mathbf{Z}^{MN}$ and $\mathbf{Z}^{M_1\cdots M_5}$ turn out to be the corresponding part of M2- and M5-branes. One finds that in order to describe the full M-algebra, we must add the charge $Z_{\boldsymbol{\alpha\beta}}$ to the superalgebra obtained from the IIB superalgebra. We do not give the unified superalgebra here, but we expect that IIB-branes and M-branes will be unified in (11+1)-dimensions.

6. Summary and discussions

We have shown that the WZ terms of Dp- and NSp-branes in the type-IIB theory are characterized by non-trivial Chevalley-Eilenberg $(p+2)$-cocycles. The total actions are equivalent to those obtained in [19] and κ-invariant. These WZ terms are shown to be constructed in a manifestly supersymmetric way by introducing a set of new spacetime superalgebras. Using these superalgebras, we have shown that Siegel's formulation generalizes to the D3-branes, the D5-branes and the NS5-branes. Namely, using the supercurrents on the supergroup manifolds corresponding to the new superalgebras, we wrote down the WZ terms, which are $(p+2)$-th order in the supercurrents ($p=5$ for the D3-branes and $p=7$ for the D5- and NS5-branes.)

The relations of these superalgebras to the M-algebra are discussed. The D5- and NS5-brane superalgebras contain generators with five spinorial indices, which cannot be related to the M-algebra. In order to construct the manifestly superinvariant WZ terms, these charges must be added. We have established the IIB-brane superalgebra with maximal "central" extension which is generated by generators: supertranslations Q_A, D-strings Σ^A, F-strings Z^A, D3-branes Σ^{ABC}, D5-branes $\Sigma^{A_1\cdots A_5}$, NS5-branes $Z^{A_1\cdots A_5}$ and KK5-branes $P^{I_1\cdots I_5}$. The superalgebra enjoys the S-duality, or generalized SO(2) R-symmetry as an automorphism of the algebra. This SO(2)-symmetry is found to be geometrically realized in twelve-dimensions with signature (11,1).

We expect that our formulation generalizes to the higher branes: KK5-, D7-, NS7-, D9- and NS9-branes. The D7- and NS7-brane charges will be contained in Σ^{ABC}, and the D9- and NS9-brane charges in Σ^A and Z^A, respectively. We have presented the IIB-brane superalgebra generated by these generators, but the construction of the corresponding WZ terms are left for the future investigations.

We have classified IIB NS/D p-branes by CE cohomology $(p+2)$-cocycles. It is interesting for us to consider whether the CE cohomology classification of KK5-branes can be achieved. One wishes to know the physical meaning of the new generators with spinorial indices. In [18], the corresponding Noether charges are obtained and found to probe the topology of the ordinary superspace. The same will be true for our models too. In [17], $\mathrm{SO}(2,1) \cong \mathrm{SL}(2,\mathbb{R})$ covariant IIB superalgebra was discovered. There, translations P_a, F-strings Z^a and D-strings Σ_a formed a $\mathrm{SO}(2,1)$ triplet. It is interesting for us to construct $\mathrm{SO}(2,1)$ covariant superalgebra which contains charges of KK5-branes, NS5-branes and D5-branes as a $\mathrm{SO}(2,1)$ triplet. If this is achieved, new geometric formulation of (p,q) 5-branes will be given.

Our formulation will be generalized to the case where the target space is curved. For example, one considers the Green-Schwarz superstring actions on anti-de Sitter spaces: $AdS_3 \times S^3$, $AdS_5 \times S^5$, etc. The corresponding coset superspaces are $(\mathrm{SU}(1,1|2)_L \times \mathrm{SU}(1,1|2)_R)/(\mathrm{SO}(2,1) \times \mathrm{SO}(3))$ for the GS strings on $AdS_3 \times S^3$ and $(\mathrm{SU}(2,2|4))/(\mathrm{SO}(4,1) \times \mathrm{SO}(5))$ for the GS strings on $AdS_5 \times S^5$. In order for our formulation to generalize to these cases, the supergroups $\mathrm{SU}(1,1|2)_L \times \mathrm{SU}(1,1|2)_R$ or $\mathrm{SU}(2,2|4)$ must be extended including new generators. If this is achieved, the new coordinates corresponding to new generators will be contained in a surface term. We hope that the physical meaning of the new coordinates will be revealed with considering these models, and our formulation will shed some light on the various string dualities.

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A. D5-brane superalgebra

The D5-brane superalgebra is generated by supertranslations Q_A, D-strings Σ^A, F-strings Z^A, D3-branes Σ^{ABC} and D5-branes $\Sigma^{A_1\cdots A_5}$. The MC equations corresponding to the D5-branes are (3.13), (3.14), (3.17), (3.22) and

$$d\Pi^D_{a_1\cdots a_5} = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_5}\sigma_1)_{\boldsymbol{\alpha\beta}}\,, \tag{A.1}$$

$$\begin{aligned} d\Pi^D_{a_1\cdots a_4\boldsymbol{\alpha}} &= \Pi^{\boldsymbol{\beta}}\Pi^D_{a_1\cdots a_4 b}\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} + 4\Pi^{\boldsymbol{\beta}}\Pi_{a_1a_2a_3}(\gamma_{a_4}\sigma_3)_{\boldsymbol{\alpha\beta}} + \\ &\quad + \Pi^{\boldsymbol{\beta}}\Pi^b(\gamma_{a_1\cdots a_4 b}\sigma_1)_{\boldsymbol{\alpha\beta}} - 4\Pi^{\boldsymbol{\beta}}\Pi_{a_1}(\gamma_{a_2a_3a_4}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \end{aligned} \tag{A.2}$$

$$\begin{aligned} d\Pi^D_{a_1a_2a_3\boldsymbol{\alpha\beta}} &= -\frac{1}{2}\Pi^D_{a_1a_2a_3bc}\Pi^b(\gamma_c 1)_{\boldsymbol{\alpha\beta}} + \frac{1}{4}\Pi^D_{a_1a_2a_3b\boldsymbol{\gamma}}\Pi^\gamma\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} + \\ &\quad + 2\Pi^D_{a_1a_2a_3b\boldsymbol{\alpha}}\Pi^\gamma\left(\gamma^b 1\right)_{\boldsymbol{\beta\gamma}} - \frac{3}{2}\Pi^b\Pi_{ba_1a_2}(\gamma_{a_3}\sigma_3)_{\boldsymbol{\alpha\beta}} + \end{aligned}$$

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$$
\begin{aligned}
&+\Pi^b\Pi_{a_1a_2a_3}(\gamma_b\sigma_3)_{\boldsymbol{\alpha\beta}}+\frac{3}{2}\Pi_{ba_1a_2}\Pi_{a_3}\left(\gamma^b1\right)_{\boldsymbol{\alpha\beta}}-\\
&-\frac{1}{2}\Pi_b\Pi_{a_1a_2a_3}\left(\gamma^b1\right)_{\boldsymbol{\alpha\beta}}+\frac{3}{4}\Pi^{\boldsymbol{\gamma}}\Pi_{a_1a_2\boldsymbol{\gamma}}(\gamma_{a_3}\sigma_3)_{\boldsymbol{\alpha\beta}}+\\
&+6\Pi^{\boldsymbol{\gamma}}\Pi_{a_1a_2\boldsymbol{\alpha}}(\gamma_{a_3}\sigma_3)_{\boldsymbol{\beta\gamma}}-3\Pi_{a_1}\Pi'_{a_2}(\gamma_{a_3}\sigma_3)_{\boldsymbol{\alpha\beta}}+\\
&+2\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\alpha}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\beta\gamma}}+\frac{1}{2}\Pi^b\Pi^c(\gamma_{a_1a_2a_3bc}\sigma_1)_{\boldsymbol{\alpha\beta}}-\\
&-3\Pi_{a_1}\Pi_{a_2}(\gamma_{a_3}\sigma_1)_{\boldsymbol{\alpha\beta}}+\frac{1}{2}\Pi^b\Pi_b(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha\beta}}-\\
&-3\Pi^b\Pi_{a_1}(\gamma_{a_2a_3b}i\sigma_2)_{\boldsymbol{\alpha\beta}}++\frac{1}{4}\Pi^{\boldsymbol{\gamma}}\Pi_{\boldsymbol{\gamma}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,,
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
d\Pi^D_{a_1a_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}=&\ 2\Pi^b\Pi^D_{a_1a_2bc\boldsymbol{\alpha}_1}(\gamma^c1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\Pi^{\boldsymbol{\beta}}\Pi^D_{a_1a_2b\boldsymbol{\beta}\boldsymbol{\alpha}_1}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+5\Pi^{\boldsymbol{\beta}}\Pi^D_{a_1a_2b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}+4\Pi_{a_1}\Pi_{a_2b\boldsymbol{\alpha}_1}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-3\Pi_b\Pi_{a_1a_2\boldsymbol{\alpha}_1}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+4\Pi^b\Pi_{ba_1\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+5\Pi^b\Pi_{a_1a_2\boldsymbol{\alpha}_1}(\gamma_b\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi^{\boldsymbol{\beta}}\Pi_{a_1\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+10\Pi^{\boldsymbol{\beta}}\Pi_{a_1\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}-3\Pi_{a_1a_2b}\Pi_{\boldsymbol{\alpha}_1}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-6\Pi'_{a_1}\Pi_{\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+4\Pi'_{\boldsymbol{\alpha}_1}\Pi_{a_1}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+5\Pi^b\Pi_{\boldsymbol{\alpha}_1}(\gamma_{a_1a_2b}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-10\Pi_{a_1}\Pi_{\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,,
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
d\Pi^D_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}=&-\Pi^b\Pi^D_{abc\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^c1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}-\frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi^D_{ab\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}-\\
&-\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi^D_{ab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}}+\Pi_a\Pi_{b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}+\\
&+2\Pi_b\Pi_{a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}+\Pi^b\Pi_{b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}-\\
&-3\Pi^b\Pi_{a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_b\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}-\frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi_{\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}-\\
&-\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}}-3\Pi_{ab\boldsymbol{\alpha}_1}\Pi_{\boldsymbol{\alpha}_2}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}+\\
&+3\Pi'_{\boldsymbol{\alpha}_1}\Pi_{\boldsymbol{\alpha}_2}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}+3\Pi_{\boldsymbol{\alpha}_1}\Pi_{\boldsymbol{\alpha}_2}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
d\Pi^D_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5}=&-\frac{35}{6}\Pi^{\boldsymbol{\beta}}\Pi^D_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\gamma^a1)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}}-\frac{5}{3}\Pi^{\boldsymbol{\beta}}\Pi^D_{a\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}+\\
&+\Pi^a\Pi^D_{ab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^b1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}+\frac{7}{2}\Pi^a\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}-\\
&-\frac{5}{2}\Pi_a\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}+\frac{15}{2}\Pi_{\boldsymbol{\alpha}_1}\Pi_{a\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}.
\end{aligned}
\tag{A.6}
$$

B. NS5-brane superalgebra

The NS5-brane superalgebra is generated by supertranslations Q_A, D-strings Σ^A, F-strings Z^A, D3-branes Σ^{ABC} and NS5-branes $Z^{A_1\cdots A_5}$. The MC equations corresponding to the NS5-branes are (3.13), (3.14), (3.17), (3.22) and

$$d\Pi^{NS}_{a_1\cdots a_5} = +\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{a_1\cdots a_5}\sigma_3)_{\boldsymbol{\alpha\beta}}\,, \tag{B.1}$$

$$\begin{aligned} d\Pi^{NS}_{a_1\cdots a_4\boldsymbol{\alpha}} =& \Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a_1\cdots a_4 b}(\gamma^b\sigma_3)_{\boldsymbol{\alpha\beta}} + 4\Pi^{\boldsymbol{\beta}}\Pi_{a_1a_2a_3}(\gamma_{a_4}\sigma_1)_{\boldsymbol{\alpha\beta}} - \\ &-\Pi^{\boldsymbol{\beta}}\Pi^b(\gamma_{a_1\cdots a_4 b}\sigma_3)_{\boldsymbol{\alpha\beta}} + 4\Pi^{\boldsymbol{\beta}}\Pi'_{a_1}(\gamma_{a_2a_3a_4}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \end{aligned} \tag{B.2}$$

$$\begin{aligned} d\Pi^{NS}_{a_1a_2a_3\boldsymbol{\alpha\beta}} =& -\frac{1}{2}\Pi^{NS}_{a_1a_2a_3bc}\Pi^b(\gamma_c 1)_{\boldsymbol{\alpha\beta}} + \frac{1}{4}\Pi^{NS}_{a_1a_2a_3b\boldsymbol{\gamma}}\Pi^{\boldsymbol{\gamma}}\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} - \\ &-2\Pi^{NS}_{a_1a_2a_3b\boldsymbol{\alpha}}\Pi^{\boldsymbol{\gamma}}\left(\gamma^b 1\right)_{\boldsymbol{\beta\gamma}} - \frac{3}{2}\Pi^b\Pi_{ba_1a_2}(\gamma_{a_3}\sigma_1)_{\boldsymbol{\alpha\beta}} + \\ &+\Pi^b\Pi_{a_1a_2a_3}(\gamma_b\sigma_1)_{\boldsymbol{\alpha\beta}} - \frac{3}{2}\Pi_{ba_1a_2}\Pi'_{a_3}\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} + \\ &+\frac{1}{2}\Pi'_b\Pi_{a_1a_2a_3}\left(\gamma^b 1\right)_{\boldsymbol{\alpha\beta}} + \frac{3}{4}\Pi^{\boldsymbol{\gamma}}\Pi_{a_1a_2\boldsymbol{\gamma}}(\gamma_{a_3}\sigma_1)_{\boldsymbol{\alpha\beta}} + \\ &+6\Pi^{\boldsymbol{\gamma}}\Pi_{a_1a_2\boldsymbol{\alpha}}(\gamma_{a_3}\sigma_1)_{\boldsymbol{\beta\gamma}} + 3\Pi'_{a_1}\Pi_{a_2}(\gamma_{a_3}\sigma_1)_{\boldsymbol{\alpha\beta}} - \\ &-2\Pi^{\boldsymbol{\gamma}}\Pi'_{\boldsymbol{\alpha}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\beta\gamma}} - \frac{1}{2}\Pi^b\Pi^c(\gamma_{a_1a_2a_3bc}\sigma_3)_{\boldsymbol{\alpha\beta}} + \\ &+3\Pi'_{a_1}\Pi'_{a_2}(\gamma_{a_3}\sigma_3)_{\boldsymbol{\alpha\beta}} - \frac{1}{2}\Pi^b\Pi'_b(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha\beta}} + \\ &+3\Pi^b\Pi'_{a_1}(\gamma_{a_2a_3b}i\sigma_2)_{\boldsymbol{\alpha\beta}} - \frac{1}{4}\Pi^{\boldsymbol{\gamma}}\Pi'_{\boldsymbol{\gamma}}(\gamma_{a_1a_2a_3}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \end{aligned} \tag{B.3}$$

$$\begin{aligned} d\Pi^{NS}_{a_1a_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} =& 2\Pi^b\Pi^{NS}_{a_1a_2bc\boldsymbol{\alpha}_1}(\gamma^c 1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a_1a_2b\boldsymbol{\beta}\boldsymbol{\alpha}_1}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\ &+5\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a_1a_2b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} - 4\Pi'_{a_1}\Pi_{a_2b\boldsymbol{\alpha}_1}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\ &+3\Pi'_b\Pi_{a_1a_2\boldsymbol{\alpha}_1}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 4\Pi^b\Pi_{ba_1\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\ &+5\Pi^b\Pi_{a_1a_2\boldsymbol{\alpha}_1}(\gamma_b\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 2\Pi^{\boldsymbol{\beta}}\Pi_{a_1\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\ &+10\Pi^{\boldsymbol{\beta}}\Pi_{a_1\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}} + 3\Pi_{a_1a_2b}\Pi'_{\boldsymbol{\alpha}_1}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + \\ &+6\Pi_{a_1}\Pi'_{\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - 4\Pi_{\boldsymbol{\alpha}_1}\Pi'_{a_1}(\gamma_{a_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} - \\ &-5\Pi^b\Pi'_{\boldsymbol{\alpha}_1}(\gamma_{a_1a_2b}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} + 10\Pi'_{a_1}\Pi'_{\boldsymbol{\alpha}_1}(\gamma_{a_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,, \end{aligned} \tag{B.4}$$

$$
\begin{aligned}
d\Pi^{NS}_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} = & -\Pi^b\Pi^{NS}_{abc\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^c 1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{ab\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{ab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} - \Pi'_a\Pi_{b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -2\Pi'_b\Pi_{a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \Pi^b\Pi_{b\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -3\Pi^b\Pi_{a\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_b\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi_{\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} + 3\Pi_{ab\boldsymbol{\alpha}_1}\Pi'_{\boldsymbol{\alpha}_2}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -3\Pi_{\boldsymbol{\alpha}_1}\Pi'_{\boldsymbol{\alpha}_2}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - 3\Pi'_{\boldsymbol{\alpha}_1}\Pi'_{\boldsymbol{\alpha}_2}(\gamma_a\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}\,, \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
d\Pi^{NS}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} = & -\frac{35}{6}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\gamma^a 1)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}} - \frac{5}{3}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{a\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\Pi^a\Pi^{NS}_{ab\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^b 1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \frac{7}{2}\Pi^a\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma_a\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\frac{5}{2}\Pi'_a\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \frac{15}{2}\Pi'_{\boldsymbol{\alpha}_1}\Pi_{a\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^a 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}\,. \qquad \text{(B.6)}
\end{aligned}
$$

C. KK5-brane superalgebra

The KK5-brane superalgebra is generated by supertranslations Q_A, D-strings Σ^A, F-strings Z^A, D3-branes Σ^{ABC}, D5-branes $\Sigma^{A_1\cdots A_5}$, NS5-branes $Z^{A_1\cdots A_5}$ and KK5-branes $P^{A_1\cdots A_5}$. The MC equations needed for the KK5-branes are (3.13), (3.14), (3.17), (3.22), (A.1)–(A.6), (B.1)–(B.6) and

$$
d\Pi^{KK}_{i_1\cdots i_5} = -\frac{1}{2}\Pi^{\boldsymbol{\alpha}}\Pi^{\boldsymbol{\beta}}(\gamma_{i_1\cdots i_5}1)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \qquad \text{(C.1)}
$$

$$
\begin{aligned}
d\Pi^{KK}_{i_1\cdots i_4\boldsymbol{\alpha}} = & \Pi^{\boldsymbol{\beta}}\Pi^j(\gamma_{ji_1\cdots i_4}1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \Pi^{\boldsymbol{\beta}}\Pi_\sharp(\gamma_{\sharp i_1\cdots i_4}\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \Pi^{\boldsymbol{\beta}}\Pi'_\sharp(\gamma_{i_1\cdots i_4\sharp}\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
& +\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{ji_1\cdots i_4}\left(\gamma^j 1\right)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp i_1\cdots i_4}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp i_1\cdots i_4}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
& +6\Pi^{\boldsymbol{\beta}}\Pi_{\sharp i_1 i_2}(\gamma_{\sharp i_3 i_4}i\sigma_2)_{\boldsymbol{\alpha}\boldsymbol{\beta}}\,, \qquad \text{(C.2)}
\end{aligned}
$$

$$
\begin{aligned}
d\Pi^{KK}_{i_1 i_2 i_3\boldsymbol{\alpha}\boldsymbol{\beta}} = & \frac{1}{2}\Pi^j\Pi^k(\gamma_{jki_1i_2i_3}1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \Pi_\sharp\Pi^j(\gamma_{ji_1i_2i_3\sharp}\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
& +\Pi'_\sharp\Pi^j(\gamma_{ji_1i_2i_3\sharp}\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{1}{2}\Pi^{KK}_{i_1i_2i_3jk}\Pi^j\left(\gamma^k 1\right)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
& +\frac{1}{2}\Pi^{NS}_{\sharp i_1i_2i_3j}\Pi_\sharp\left(\gamma^j 1\right)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \frac{1}{2}\Pi^{D}_{\sharp i_1i_2i_3j}\Pi'_\sharp\left(\gamma^j 1\right)_{\boldsymbol{\alpha}\boldsymbol{\beta}} + \\
& +\frac{1}{2}\Pi^{NS}_{\sharp i_1i_2i_3j}\Pi^j(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{1}{2}\Pi^{D}_{\sharp i_1i_2i_3j}\Pi^j(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \\
& -\frac{1}{2}\Pi_{i_1i_2i_3}\Pi'_\sharp(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}\boldsymbol{\beta}} - \frac{1}{2}\Pi_{i_1i_2i_3}\Pi_\sharp(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}\boldsymbol{\beta}} +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{4}\Pi^{KK}_{i_1i_2i_3j\boldsymbol{\gamma}}\Pi^{\gamma}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha\beta}}+\frac{1}{4}\Pi^{NS}_{\sharp i_1i_2i_3\boldsymbol{\gamma}}\Pi^{\gamma}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha\beta}}-\\
&-\frac{1}{4}\Pi^{D}_{\sharp i_1i_2i_3\boldsymbol{\gamma}}\Pi^{\gamma}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha\beta}}-2\Pi^{KK}_{ji_1i_2i_3\boldsymbol{\alpha}}\Pi^{\gamma}\left(\gamma^{i}1\right)_{\boldsymbol{\beta\gamma}}+\\
&+2\Pi^{NS}_{\sharp i_1i_2i_3\boldsymbol{\alpha}}\Pi^{\gamma}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\beta\gamma}}-2\Pi^{D}_{\sharp i_1i_2i_3\boldsymbol{\alpha}}\Pi^{\gamma}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\beta\gamma}}-\\
&-3\Pi'_{\sharp}\Pi_{\sharp i_1i_2}(\gamma_{i_3}\sigma_3)_{\boldsymbol{\alpha\beta}}-3\Pi_{\sharp}\Pi_{\sharp i_1i_2}(\gamma_{i_3}\sigma_1)_{\boldsymbol{\alpha\beta}}-\\
&-6\Pi^{\gamma}\Pi_{\sharp i_1\boldsymbol{\alpha}}(\gamma_{\sharp i_2i_3}i\sigma_2)_{\boldsymbol{\beta\gamma}}+\frac{3}{2}\Pi_{\sharp ji_1}\Pi_{\sharp i_2i_3}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha\beta}}+\\
&+\frac{3}{2}\Pi'_{i_1}\Pi_{\sharp i_2i_3}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha\beta}}+\frac{3}{2}\Pi_{i_1}\Pi_{\sharp i_2i_3}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha\beta}}+\\
&+\Pi'_{\sharp}\Pi_{\sharp}(\gamma_{i_1i_2i_3}i\sigma_2)_{\boldsymbol{\alpha\beta}}-\frac{3}{2}\Pi^{j}\Pi_{\sharp ji_1}(\gamma_{i_2i_3\sharp}i\sigma_2)_{\boldsymbol{\alpha\beta}}+\\
&+\frac{3}{2}\Pi_{\sharp}\Pi'_{i_1}(\gamma_{i_2i_3\sharp}i\sigma_2)_{\boldsymbol{\alpha\beta}}-\frac{3}{2}\Pi'_{\sharp}\Pi_{i_1}(\gamma_{i_2i_3\sharp}i\sigma_2)_{\boldsymbol{\alpha\beta}}+\\
&+3\Pi^{j}\Pi_{\sharp i_1i_2}(\gamma_{i_3j\sharp}i\sigma_2)_{\boldsymbol{\alpha\beta}}-\frac{3}{4}\Pi^{\gamma}\Pi_{\sharp i_1\boldsymbol{\gamma}}(\gamma_{i_2i_3\sharp}i\sigma_2)_{\boldsymbol{\alpha\beta}}\,, \qquad \text{(C.3)}
\end{aligned}
$$

$$
\begin{aligned}
d\Pi^{KK}_{i_1i_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3} =\; &2\Pi^{i}\Pi^{KK}_{i_1i_2jk\boldsymbol{\alpha}_1}\left(\gamma^{k}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi_{\sharp}\Pi^{NS}_{\sharp i_1i_2j\boldsymbol{\alpha}_1}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+2\Pi'_{\sharp}\Pi^{D}_{\sharp i_1i_2j\boldsymbol{\alpha}_1}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi^{j}\Pi^{NS}_{\sharp i_1i_2j\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-2\Pi^{j}\Pi^{D}_{\sharp i_1i_2j\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi'_{\sharp}\Pi_{i_1i_2\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+2\Pi_{\sharp}\Pi_{i_1i_2\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{i_1i_2j\boldsymbol{\beta}\boldsymbol{\alpha}_1}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp i_1i_2\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp i_1i_2\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+5\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{i_1i_2j\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}-5\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp i_1i_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}+\\
&+5\Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp i_1i_2\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}+10\Pi'_{\sharp}\Pi_{\sharp i_1\boldsymbol{\alpha}_1}(\gamma_{i_2}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+10\Pi_{\sharp}\Pi_{\sharp i_1\boldsymbol{\alpha}_1}(\gamma_{i_2}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+5\Pi^{\boldsymbol{\beta}}\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_{\sharp i_1i_2}i\sigma_2)_{\boldsymbol{\alpha}_3\boldsymbol{\beta}}+\\
&+6\Pi_{\sharp ji_1}\Pi_{\sharp i_2\boldsymbol{\alpha}_1}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+6\Pi'_{i_1}\Pi_{\sharp i_2\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+6\Pi_{i_1}\Pi_{\sharp i_2\boldsymbol{\alpha}_1}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi_{\sharp j\boldsymbol{\alpha}_1}\Pi_{\sharp i_1i_2}\left(\gamma^{i}1\right)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-2\Pi'_{\boldsymbol{\alpha}_1}\Pi_{\sharp i_1i_2}(\gamma^{\sharp}\sigma_3)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-2\Pi_{\boldsymbol{\alpha}_1}\Pi_{\sharp i_1i_2}(\gamma^{\sharp}\sigma_1)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-10\Pi^{j}\Pi_{\sharp i_1\boldsymbol{\alpha}_1}(\gamma_{i_2j\sharp}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-2\Pi^{j}\Pi_{\sharp j\boldsymbol{\alpha}_1}(\gamma_{\sharp i_1i_2}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}-\\
&-2\Pi_{\sharp}\Pi'_{\boldsymbol{\alpha}_1}(\gamma_{\sharp i_1i_2}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+2\Pi'_{\sharp}\Pi_{\boldsymbol{\alpha}_1}(\gamma_{\sharp i_1i_2}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}+\\
&+\Pi^{\boldsymbol{\beta}}\Pi_{\sharp\boldsymbol{\beta}\boldsymbol{\alpha}_1}(\gamma_{\sharp i_1i_2}i\sigma_2)_{\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\,, \qquad \text{(C.4)}
\end{aligned}
$$

$$
\begin{aligned}
d\Pi^{KK}_{i\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4} = {} & -\Pi^j\Pi^{KK}_{ijk\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^k 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \Pi^j\Pi^{NS}_{\sharp ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\Pi^j\Pi^{D}_{\sharp ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \Pi_\sharp\Pi^{NS}_{\sharp ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \\
& +\Pi'_\sharp\Pi^{D}_{\sharp ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \Pi'_\sharp\Pi_{i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\Pi_\sharp\Pi_{i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{ij\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp i\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \frac{3}{10}\Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp i\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} - \frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} + \\
& +\frac{6}{5}\Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\beta}} + 3\Pi'_\sharp\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_i\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \\
& +3\Pi_\sharp\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_i\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + 2\Pi_{\sharp ij}\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -2\Pi'_i\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - 2\Pi_i\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} + \\
& +3\Pi_{\sharp j\boldsymbol{\alpha}_1}\Pi_{\sharp i\boldsymbol{\alpha}_2}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - 3\Pi'_{\boldsymbol{\alpha}_1}\Pi_{\sharp i\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - \\
& -3\Pi_{\boldsymbol{\alpha}_1}\Pi_{\sharp i\boldsymbol{\alpha}_2}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4} - 3\Pi^j\Pi_{\sharp\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2}(\gamma_{\sharp ij}i\sigma_2)_{\boldsymbol{\alpha}_3\boldsymbol{\alpha}_4}\,, \qquad \text{(C.5)} \\
d\Pi^{KK}_{\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_5} = {} & -\frac{3}{5}\Pi^i\Pi^{KK}_{ij\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}\left(\gamma^i 1\right)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \frac{3}{5}\Pi^i\Pi^{NS}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\frac{3}{5}\Pi^i\Pi^{D}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \frac{3}{5}\Pi_\sharp\Pi^{NS}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^i 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \\
& -\frac{3}{5}\Pi'_\sharp\Pi^{D}_{\sharp i\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^i 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \frac{3}{5}\Pi'_\sharp\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \\
& -\frac{3}{5}\Pi_\sharp\Pi_{\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \Pi^{\boldsymbol{\beta}}\Pi^{KK}_{i\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^i 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} - \\
& -\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp\boldsymbol{\beta}\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +\frac{7}{2}\Pi^{\boldsymbol{\beta}}\Pi^{KK}_{i\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\gamma^i 1)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}} - \frac{7}{2}\Pi^{\boldsymbol{\beta}}\Pi^{NS}_{\sharp\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}} + \\
& +\frac{7}{2}\Pi^{\boldsymbol{\beta}}\Pi^{D}_{\sharp\boldsymbol{\alpha}_1\cdots\boldsymbol{\alpha}_4}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_5\boldsymbol{\beta}} - 6\Pi_{\sharp i\boldsymbol{\alpha}_1}\Pi_{\sharp\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^i 1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + \\
& +6\Pi'_{\boldsymbol{\alpha}_1}\Pi_{\sharp\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_3)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5} + 6\Pi_{\boldsymbol{\alpha}_1}\Pi_{\sharp\boldsymbol{\alpha}_2\boldsymbol{\alpha}_3}(\gamma^\sharp\sigma_1)_{\boldsymbol{\alpha}_4\boldsymbol{\alpha}_5}\,. \qquad \text{(C.6)}
\end{aligned}
$$

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